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13 ABSTRACT

After a brief presentation of the salient observational facts about tropical cyclones, the two principal models for the cyclone-scale structure and maintenance of the quasisteady mature stage are described--the older model by Riehl and Malkus and the recently developed model by Carrier. One point of disagreement between the two models is that Riehl and Malkus postulate that greatly augmented sea-to-air transfer of latent and sensible heat is required to sustain a hurricane, while Carrier states that (because the storm convects a large mass of warm moist air with it) the local sea-to-air transfer of total enthalpy is about the same whether a hurricane is present or not. Analyses relating to maximum swirl speed estimates and to surface-frictional-layer dynamics and energetics are reviewed to support the Carrier model. Reservations concerning both the frictional inflow layer modeling and also the interpretation of temperature measurements by Riehl and Malkus are stated in detail. Since the cyclone-scale structure of a fully developed tropical cyclone is now believed to be correctly sorted out by Carrier's model, attention is then turned to refinements of Carrier's earlier outline of tropical-cyclone intensification, during transition from depression to hurricane. (Continued)

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TROPICAL CYCLONES

by

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SUMMARY

After a brief presentation of the salient observational facts about tropical cyclones, the two principal models for the cyclone-scale structure and maintenance of the quasisteady mature stage are described--the older model by Riehl and Malkus and the recently developed model by Carrier. One point of disagreement between the two models is that Riehl and Malkus postulate that greatly augmented sea-to-air transfer of latent and sensible heat is required to sustain a hurricane, while Carrier states that (because the storm convects a large mass of warm moist air with it) the local sea-to-air transfer of total enthalpy is about the same whether a hurricane is present or not. Analyses relating to maximum swirl speed estimates and to surface-frictional-layer dynamics and energetics are reviewed to support the Carrier model. Reservations concerning both the frictional inflow layer modeling and also the interpretation of temperature measurements by Riehl and Malkus are stated in detail. Since the cyclone-scale structure of a fully developed tropical cyclone is now believed to be correctly sorted out by Carrier's model, attention is then turned to refinements of Carrier's earlier outline of tropical-cyclone intensification, during transition from depression to hurricane. Intensification theory necessitates careful consideration of the problem that has preoccupied most numerical modelers--parameterization of cumulus convection. Here some preliminary and tentative modeling of the development of a warm core is attempted, in which organized convection lifts and ejects initially present air, whose thermodynamic state is maintained by relatively slow ambient processes (cumulus convection, turbulent mixing, radiational cooling). More basic incorporation of Charney's concepts concerning "conditional instability of the second kind" on the cumulus scale, just as Carrier's model in a sense already utilizes the concepts on a cyclone scale, is cited as probably the source of the next important progress in modeling of tropical cyclone intensification.

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I. INTRODUCTION

I.A MOTIVATIONS FOR HURRICANE MODELING

In an average year the Atlantic and Gulf coast states suffer over \$100 million damage and 50-100 fatalities owing to hurricanes; in a severe year damage will exceed \$1 billion (Meyer 1971). Hurricane Camille alone in August 1969 cost about 500 lives and \$1 billion in destruction (De Angelis 1969). The threat comes in the form of wind [over 200 mph is known (Lear 1969)]; in rainfall [27 in. in 24 hours (Schwarz 1970)--since 1886 hurricanes have caused over 60 floods in the U.S. (Alaka 1968)]; and in storm surges [coastal ocean levels have risen as much as 15-20 feet (Alaka 1968)]. On the positive side is the fact that on a long-term basis the average hurricane-associated rainfall over the Eastern states is a substantial fraction (perhaps one-third) of the total rainfall; without it, droughts seem inevitable.

The threat to execution of military missions is evident; there are annually about fifty hurricanes on a global basis, affecting all oceans except the S. Atlantic (Palmén and Newton, 1969). Actually unnecessary preparations by DOD installations owing to false hurricane and typhoon warnings is annually about \$8.3 million--this is aside from diversion-of-manpower costs (Malone and Leimer 1971). In path predictions for either military or civilian use, three times the area actually hit by a tropical cyclone is typically placed under hurricane warnings (Meyer 1971; Malone and Leimer 1971).

These considerations alone would justify an effort at understanding the genesis, steady-state structure, and decay of tropical cyclones to be able to forecast their onset, intensity, and path. However, there is also an additional motivation of particular relevance to the current study. A hurricane has been estimated to have a kinetic energy comparable to that of a hydrogen bomb (10^{10} kilowatt-hours) (Battan 1961) and hurricanes have

rained over ninety-five inches over a spot in four days (Silver Hill, Jamaica in November, 1909) (Alaka 1968). The point is that a system with this much energy and water substance is no local accident, but could be appreciable to global balances.* There are several efforts in the U.S. alone on global atmospheric simulation on high-speed, large-storage digital computers for both short-term weather prediction and also long-term climatological reconstruction and anticipated evolution.† The effort at the NOAA Geophysical Fluid Dynamics Laboratory in Princeton, New Jersey includes a program under Kurihara (1971) devoted to numerical integration of the basic equations believed to model a tropical cyclone. Similarly, a major motivation behind the TRW tropical cyclone project being described here is its possible contribution to the ARPA-sponsored global climatological analysis under way at Rand Corporation in Santa Monica, California. While there are other familiar rapidly swirling vortices occurring in the lower atmosphere (tornadoes, waterspouts, dustdevils, firewhirls, and swirls attending volcanic eruptions), none of these persist long enough in time and occur over a large enough area to warrant consideration for incorporation in current feasible global-scale computations. On the fastest conventional computers a grid no finer than 4° x 5° is practical for the Rand program, and even on an advanced computer like the Illiac IV no grid finer than 1° x 1° seems practical (Rapp 1970).

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*Because hurricanes have radial extents of several hundred miles and extend to the tropopause, and because hurricanes persist for weeks, it seems implausible that they are mere accidents. The fact that they occur annually, mainly in the autumn after the long summer heating of the tropical oceans by solar radiation, suggests the speculation that they are a mechanism for relaxing energy poleward when the usual Hadley-cell mechanism is not sufficient. Hurricanes in general do turn poleward after drifting westward in the trades.

†One effort is carried on at the Geophysical Fluid Dynamics Laboratory of NOAA on the Forrestal Campus of Princeton University in New Jersey; this effort, under Dr. J. Smagorinsky, has Department of Commerce funding (Smagorinsky 1963). A second effort is conducted by Dr. A. Kasahara under NSF funding at Boulder, Colorado (Kasahara and Washington 1967). The third effort is supervised by Dr. R. Rapp of Rand Corp. in Santa Monica, California under ARPA (Department of Defense) funding; this effort concerns solution of the Mintz-Arakawa model on the Illiac IV computer (Rapp 1970). The goals and techniques of these three efforts differ appreciably.

I.B SOME OBSERVATIONAL FACTS ON HURRICANES

Every year, especially in the late summer and early fall, several of the many very large depressions in the trades (specifically, disturbances between 5° and 15° of the equator) intensify into tropical cyclones* [winds in excess of 74 mph by convention] over warm tropical oceans (usually at least 26°C to 27°C, often 28°C and higher). These tropical cyclones are typically a thousand miles in diameter and no more than ten miles in height; they often persist from one to several weeks, traveling westward in the barotropic trades at about twenty miles per hours before moving poleward at greater translational speeds. These vortical storms are cyclonic in the Northern Hemisphere and anticyclonic (North Pole reference) in the Southern Hemisphere, and take many days to intensify--indicating that the small rate of rotation of the earth is the source of angular momentum (Palmén and Newton 1969). In fact, conservation of angular momentum in itself indicates that a fluid particle in the tropics drawn in about five hundred miles will swirl at several hundred miles per hour. Tropical cyclones have local designations around the globe (e.g., hurricanes in the North Atlantic, typhoons in the northwestern Pacific, papagallos on the west coast of Central America, baguios in the Philippine Islands, willy-willies in Australia, cyclones in the northern Indian Ocean, and trovados near Madagascar).

Tropical cyclones, which total about fifty in a typical year, are known over all oceans except the South Atlantic. With satellite photography the inspection of broad ocean expanses has improved; it seems that there are about eight tropical storms annually in the North Atlantic, and

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*"A tropical cyclone starts out as a tropical disturbance in which there is a slight surface circulation and perhaps one closed isobar. When the wind increases to about 20 knots and there is more than one closed isobar around the center, it is called a tropical depression. When the wind rises to more than 34 knots, and there are several closed isobars, it becomes known as a tropical storm. If the winds exceed 64 knots (74 miles/hour), it is classified as a hurricane or typhoon or cyclone (depending on location)" (Day 1966, p. 187).

about half[†] of these intensify into tropical cyclones (Meyer 1971). Many Atlantic hurricanes can be traced to biweekly disturbances that begin as sandstorms over the Sahara (Palmén and Newton 1969); these cyclonic depressions may extend 1000 nautical miles and drift westward at up to ten knots. Analysis of dust samples taken on Caribbean isles after hurricane passage reportedly confirms this (Jennings 1970). As the peak of the North Atlantic hurricane season approaches, the region where tropical storms reach hurricane intensity moves eastward from the Gulf of Mexico and the Caribbean to the Cape Verde Islands; as the hurricane season passes, the spawning ground moves westward again to the Caribbean (Meyer 1971).

It is known that hurricanes form where there is sustained local convective activity over warm tropical seas (so that air lifted on a moist adiabat remains warmer than the undisturbed ambient up to 12 km), where there is enhanced cyclonic shear (as occurs when the Intertropical Convergence Zone lies at a considerable distance from the equator), and where there is weak vertical shear. The last requirement supposedly explains the anomalous cyclone season for the northern Indian Ocean, with twin-peak frequencies of occurrence (in spring and fall) with a relatively uneventful summer season (Palmén and Newton 1969). On the other hand, hurricanes tend to weaken over land moderately rapidly [the central pressure of Camille rose from 905 to 990 mb in about thirteen and one-half hours after land fall (Bradbury 1971)].

Tropical cyclones have a structure characterized in the mature stage by a relatively cloud-free calm (winds usually well below fifteen mph) eye of about ten to twenty mile radius. The eye is characterized by low pressure at sea level (often below 960 mb) and high temperatures aloft (10°C above ambient). The eye is surrounded by an eyewall, a ten-mile wide

[†] "During the warmer months, at least one easterly wave is present almost every day over the Atlantic. In that region an average of only eight disturbances per year reach tropical storm intensity . . . and about 60 percent of these achieve hurricane force . . . Thus a weak disturbance has a poor chance of becoming a tropical storm, but one that has achieved tropical storm intensity has an excellent prospect of becoming a full hurricane" (Palmén and Newton 1969, p. 503). There appears to be no criterion such that once a developing depression exceeds it, it will definitely become a hurricane.

annulus of intense convection, torrential rainfall, and deep, thick cloudiness. Outside the eye are convective rainbands that appear like pinwheels or logarithmic spirals in some satellite photographs and/or radar displays taken from above the storm. The principal velocity component in much of the storm is azimuthal (or tangential); the vertical velocity component is appreciable in the eyewall. There is low-level cyclonic inflow and (in the outer regions) high-level anticyclonic outflow (relative to an observor rotating with the earth) for a Northern Hemisphere tropical cyclone (Palmén and Newton 1969). While there is much spray, the lowest few hundred feet (at least) of the inflow layer remain cloud-free in as far as the eyewall (Riehl 1954).

It is often agreed that there is slow downward motion in the eye and in the outer regions of the storm; that the latent heat of condensation reduces the density in the eyewall to establish a large radial pressure deficit relative to ambient conditions, from hydrostatic considerations; that a still further pressure deficit from ambient occurs in the eye owing to roughly dry adiabatic recompression of air that has risen along a moist adiabat in the eyewall; and that a cyclostrophic balance (balance of radial pressure gradient and centrifugal force) yields a good estimate of the swirl speeds.

Cloud cells in the eyewall are typically 5 to 20 km thick and the smaller ones can rain over 6 in./hr. Spiral bands out to 150 km yield 0.4 in/hr, and further out, 0.1 in./hr--though 1 km convective cells can give much heavier rainfall. In the low rainfall area the precipitation is probably snow that turns to rain at the melting level (Meyer 1971). In addition to the rainfall, storm surge, and large waves already mentioned, tropical cyclones can spawn tornadoes and waterspouts (Orton 1970). Only near the center are tropical cyclones axisymmetric to good approximation; near the outer edges there is asymmetry. The location of most spawned tornadoes, maximum rainfall, the most important rainbands, and highest winds suggest that, for North Atlantic hurricanes, the storm is most severe in the right forward quadrant, with respect to an observor looking along the direction of translation (Hawkins 1971). Clearly the additive translational

contribution to the azimuthal winds is the plausible explanation of the asymmetry in wind speeds (Riehl 1954, p. 290).

Most of these remarks concern the mature hurricane and its decay. Much of what is written about intensification is tentative, nebulous, and labyrinthine. One point about which there is obvious indecision is the role of the Intertropical Convergence Zone (or Trade Confluence) in tropical cyclone genesis. Palmén and Newton (1969, p. 503) cite one source (Dunn and Miller 1969) which states that only about one-sixth of ". . Atlantic hurricanes originate as perturbations moving away from the trade-confluence zone in the Panama region . . .", while another source (Palmén and Newton 1969, p. 476) states that "the great majority of the cyclones of the tropical north Pacific form in latitudes south of 6°N but intensify rapidly only in the zone 6°N to 15°N." The Dunn and Miller book, in de-emphasizing the role of the doldrums, is at variance with both earlier and later work; for example, Byers (1944, pp. 425-428) notes that tropical cyclogenesis occurs only in those locales and during those seasons in which the Intertropical Convergence Zone (ITCZ) has been displaced far from the equator--up to 15°. Often, when the ITCZ has been displaced this far from the equator, below about 6000 ft, between the ITCZ and the equator, there is a westerly flow within the general tropical easterly flow--a possible source of enhanced shear. Byers attributes the absence of hurricanes in the South Atlantic largely to the failure of the ITCZ to become displaced south of the equator, even in February. More recently, Charney (1971) notes that the same conditions which are important to tropical cyclogenesis (low-level convergence, large moisture content and appreciable Coriolis force) and the same processes active in tropical cyclogenesis (conditional instability of the second kind--CISK) are also important in the generation of the ITCZ. The CISK process is the feeding of convective activity in a swirling flow by frictionally induced inflow in a surface boundary layer; the convection so sustained by the low-level moist inflow results in a radial pressure gradient (through local lightening of the air by condensational heat release), such that swirling is sustained to create more inflow. This important subject will be addressed again later. One other

remark about intensification is that most theories propose that an eye is formed gradually as the tropical disturbance grows into a tropical cyclone (Palmén and Newton 1969).

Path prediction for tropical-cyclones is still in an imperfect state. The prevailing method is largely historical--what did previous similar hurricanes do in similar circumstances? Early analytical work sought a "steering level" in the ambient winds [Byers (1944, p. 447) states that "... the circulation in the upper air such as at 10,000 ft., determines the hurricane path with considerable accuracy."]. Later a "steering layer" concept was found more satisfactory [Riehl (1954, p. 345) states that "... tropical storms move in the direction and with the speed of the steering current, which is defined as the pressured weighted mean flow from the surface to 300 mb over a band 8° latitude in width and centered on the storm"]. Still more recent work averages over the depth of the troposphere from 100 to 1000 mb (Sanders and Burpee 1968). There are many special circumstances; for example, coexistent binary tropical cyclones in the same hemisphere rotate about one another (Fujiwhara effect) (Brand 1970), while binary systems in different hemispheres tend to move parallel (Cox and Jager 1969)--as suggested by classical potential theory for line vortices. Prior passage of a previous cyclone can also have an effect (Brand 1971). Tropical cyclones often recurve eastward at midlatitudes along the western side of high-pressure cells, and can interact with extratropical cyclones (Palmén and Newton 1969).

II. MODELS OF A TROPICAL CYCLONE

II.A INTRODUCTION

In sorting out the thermohydrodynamics of a tropical cyclone, one is faced with understanding the interaction of two scales of phenomena. One scale is the larger cyclone scale; a mature intense tropical cyclone may easily reach radial proportions of five-hundred to one-thousand miles before the winds subside to ambient. The smaller cumulus scale is at least two, often three, orders of magnitude smaller. The cyclone must feed the cumulus scale, which in turn sustains the cyclone scale, in a cooperative interdependence.

Before turning to this interaction problem, it is probably worthwhile to categorize most current research in terms of these two scales. Almost all current theoretical research on tropical cyclones concentrates on parameterizing the cumulus convection; only by so doing can the hurricane be properly described on a high-speed digital computer. According to most workers, the gross cyclone-scale thermohydrodynamics has already been essentially and correctly outlined by Riehl and Malkus in a series of articles and books in the late 1950's and early 1960's (Riehl 1954; Malkus 1958; Malkus and Riehl 1960). A divergent point of view about cyclone-scale thermohydrodynamics has been set forth by Carrier and his co-workers in a series of articles published within the last two years (Carrier 1970; Dergarabedian and Fendell 1970; Dergarabedian and Fendell 1971; Carrier, Hammond, and George 1971; Carrier 1971a; Carrier 1971b). These articles delineate overall dynamics and thermodynamics, and imply (in contrast to Riehl-Malkus) no major augmentation of ambient heat/moisture transfer from the ocean is needed to explain hurricanes.

The controversy between the Riehl-Malkus and Carrier theories about cyclone-scale thermohydrodynamics has been briefly discussed before, but shall be reviewed here for two reasons. First, recent publications have

clarified the disagreements particularly clearly. Second -- and most important -- there seems a strong possibility that the same physical processes operating on the cyclone scale are also operating (properly scaled down) on the cumulus scale. This makes it all the more important to appreciate the structure and processes operative in a tropical cyclone in the large.

II.B THE CARRIER MODEL

In this section, mainly the structure of a mature, fully developed tropical cyclone, taken to be adequately modeled as quasisteady and axisymmetric, will be studied. In later sections transient analysis aimed at establishing how such a severe vortical storm is established will be undertaken.

The Carrier model, on the basis of subdividing the tropical cyclone into segments where different processes and scales predominate, is a four-part analysis. The four regions, indicated in Figure II.B.1, are the throughput supply I, the frictional boundary layer II, the eyewall and efflux region III, and the eye IV. Some of this subdivision is conventional, some not. Besides clarifying locally dominant physical processes, it permits retention of the minimal number of terms in locally valid quantitative formulation; this procedure simplifies the mathematical solution in a manner unavailable to any direct finite-differencing of uniformly valid equations.

The Carrier model is closed for convenience -- there is no very significant amount of mass convected across any boundary. The cylindrical-like volume encompassing the entire storm has the sea surface for its bottom; its sides lie far enough from the center (about 500 to 1000 mi) so that the winds are virtually reduced to ambient, and the swirl relative to the earth is taken as zero. The top of the storm is taken to be that height at which sea-level air in the outer part of the storm, if lifted rapidly so that the total enthalpy of a fluid particle remained constant because relatively slow ambient-maintaining processes would not have time to act, would no longer be unstable relative to the local ambient air. Such sea-level air would, of course, rise dry adiabatically until saturated;

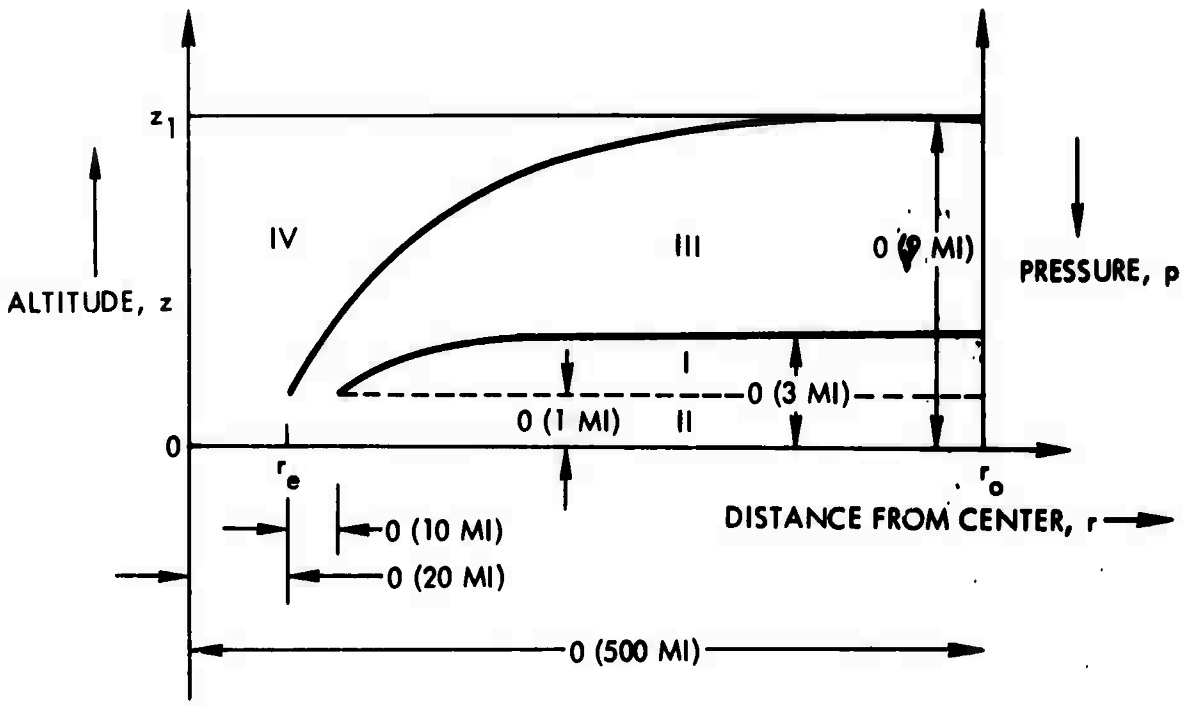


Fig. II.B.1 This conjectured configuration of a mature hurricane with rough order-of-magnitude dimensions is not drawn to scale. The subdomains are: I, throughput supply, a region of rapid swirl and slow downdraft; II, frictional boundary layer; III, eyewall; and IV, eye. Across the boundary layer II there is about a 100 mb drop and across I, a further 200 mb drop; at the top of the hurricane the pressure is about 150 mb, i.e., the top is near the tropopause.

thereafter, enough precipitation would fall out to leave the air just saturated at the local temperature and pressure, the air retaining the latent heat of phase transition. Such a locus of states is conventionally referred to as the moist adiabat. The vertical profile of the equivalent potential temperature in a typical autumnal tropical ambient is such that it decreases with increasing height to roughly 330°K at 650 mb, then increases and recovers its sea-level value of about 350°K at about 150 mb. The autumnal tropical ambient is conditionally unstable in that a particle at any height lifted dry adiabatically attains a potential temperature lower than that of the tropical ambient and hence would return to its initial position; but any sea-level particle displaced vertically enough for the onset of condensation would continue to rise many kilometers. This "instability lid" lies at so great a height that there is virtually negligible swirl, as explained below; the ambient pressure and temperature at this height is taken to describe all radial positions at this height, from the center to the outer edge. Thus, the top of the storm is an isothermal, isobaric lid with no water vapor content for current purposes.

The relatively slow processes that maintain the tropical ambient, to which allusion has just been made, are cumulus convection, turbulent mixing, and radiational transfer. While these are qualitatively easy to describe, precise quantitative formulation is very formidable.

Discussion now turns to describing each of the four regions comprising the tropical cyclone in some detail.

In region I there is warm moist air typical in stratification of the ambient atmosphere in which the hurricane was generated. This air spun up under conservation of angular momentum as it moved in toward the axis of symmetry during the formative stage. As the mature stage was approached, a gradient-wind balance of pressure, Coriolis, and centrifugal forces choked off any further inflow; the inflow is only enough to prevent the eyewall III from diffusing outward, and that requires only an exceedingly small radial flow. The air in I, then, is rapidly swirling, the azimuthal velocity component greatly increasing and the pressure greatly decreasing from the edge to the center. Under such a radial profile for the swirl,

there is a small downflux from the throughput supply I into the frictional boundary layer II. The small downflux leads to a large net mass flux into the frictional layer because of the area involved. Furthermore, the downdraft is only a gross average because locally and transiently there is intense convective activity by which clouds form and rain falls. The clouds are strained by the rapid swirl into the spiral bands seen on radar screens or in satellite photographs. The spiral bands give visualization to parts of the strain pattern, rather than streamline pattern.

In the frictional boundary layer II, the only region in which angular momentum is not conserved but is partially lost to the sea, there is appreciable influx. In fact, the azimuthal and radial velocity components are of comparable magnitude; typically, for fixed radial position, the maximum inflow speed at any axial position in the boundary layer is about one-third the maximum azimuthal speed. [This fraction is about the one reported by Hughes (1952) from flight penetration of hurricanes at altitudes of 1000 feet or less.] The vertical velocity component is much smaller. The reason for the inflow is, of course, that the no-slip boundary condition reduces the centrifugal acceleration, and a relatively uncompensated pressure gradient drives the fluid toward the axis of symmetry (so-called "tea cup effect"). Far from the axis in II the classical balance of the linear Ekman layer (friction, pressure, and Coriolis forces) suffices; since the downdraft from I to II is probably fairly independent of radial position (especially far from the eyewall) for swirl distributions of practical interest, three-quarters of the flux inward through II comes from downflux across the interface between II and I where $(r_o/2) < r < r_o$, where r_o is the radial extent of the storm. Closer in to the axis the nonlinear accelerations, especially the radial acceleration of radial and tangential momentum and centrifugal acceleration, must enter.

As the pressure gradient in I lets more air sink into II, the influx in II drives the boundary layer air moderately rapidly up a cloudy eyewall III. In the eyewall hydrostatic and cyclostrophic approximations hold; the locus of thermodynamic states is the moist adiabat based on sea-level conditions in the eyewall. The swirl near the top of III is so reduced

that, in the outflow, the air seems to an observor on earth to be rotating opposite in sense to the rotation in I and II (which is cyclonic in the Northern Hemisphere and anticyclonic in the Southern). The air in III slips over the air in I with no interaction; there is no large radial pressure gradient in much of III, unlike I.

At low altitudes the eyewall flushes moist air out of IV, and at high altitude rained-out air is entrained into IV. In time the eye becomes better defined; it is the central core in which relatively dry air sinks, is warmed by compression, and is entrained out into the eyewall or recirculated within the eye (stagnation at the base of the eye would permit transport processes to cool the eye). The relatively light eye permits much greater pressure deficits from ambient and hence supports much higher swirling speeds. At a fixed altitude, the density in the eye is less than that in the eyewall, which in turn is less than that of the ambient gas at the storm edge. Since the hydrostatic approximation is uniformly valid, spatially and temporally, in a hurricane, the sea-surface eye pressure is less than the sea-surface eyewall pressure, which is less than the sea-surface ambient pressure.

Carrier's model thus pictures the tropical cyclone as a once-through process in which a "fuel supply" -- the warm moist air in I, a part of the tropical cyclone at its inception and convected with the storm -- is slowly exhausted. The storm weakens because the air drifting down into the frictional layer toward the end is typical of the higher tropical environment and hence of lower equivalent potential temperature. Eventually the fuel supply is exhausted, and the boundary between III and I sinks toward II. Some models (e.g., Eliassen and Kleinschmidt 1957) picture a recirculation through the storm of outflow air; the storm does not survive long enough for this, nor could such air maintain the storm.

There is one important omission to the foregoing description that has been intentionally deferred: the energetics of the surface frictional layer and associated questions of air/sea transfer of latent and sensible heat. The relevant quantity to consider is the total stagnation enthalpy (the sum of static enthalpy, the heat associated with condensible

moisture, gravitational potential energy, and kinetic energy contributions). This quantity (a generalization of the equivalent potential temperature) is conserved at roughly its ambient stratification throughout regions I and II; therefore, it is described by a profile that decreases with height from 1000 mb to 650 mb, and then increases with height, as mentioned above. The implication is that the heat and mass transfer from the ocean to the atmosphere is about the same within the hurricane as in the ambient. This transfer helps compensate for the rain-out in the spiral bands and helps maintain the warm, moist nature of the air in I. [Occasionally the Carrier model is still grossly misrepresented as proposing adiabatic conditions (constant total stagnation enthalpy in II so the net heat and mass transfer from sea to air is zero); such a solution cannot possibly satisfy the parabolic boundary-value problem describing the energetics of the frictional boundary layer because it obviously violates the initial condition at $r = r_0$, the outer edge. In fact, if the supplemental flux from the ocean is entirely eliminated, as from passage over land, the spin-down time is $0(a/\nu^{1/2}\Omega^{1/2})$ where the eddy viscosity $\nu \doteq 10^{-2}$ mi^2/hr, the normal component of the rotation of the earth $\Omega \doteq 2 \times 10^{-1}$ hr^{-1}, and the height of the throughput supply $a \doteq 1$ mi -- so the spin-down time is half a day to two days.] The model of total stagnation enthalpy fixed at its ambient stratification breaks down in the eyewall III; there the vertical velocity component is at least one, probably two orders of magnitude larger than the relatively small downdrift into the boundary layer; the result is that convection dominates the slow ambient-sustaining processes so the total stagnation enthalpy is virtually constant at its sea-level value, which is roughly its ambient sea-level value.

Three specific analyses carried out by Carrier and his co-workers to corroborate aspects of this quasisteady model of the mature tropical cyclone are now briefly reviewed. These involve maximum swirl speed estimation, the dynamics of a nonlinear Ekman layer, and the energetics of the surface frictional layer. These problems are not difficult to formulate nor, for the accuracy of result required, are they difficult to solve. Much novel and valuable information about tropical cyclones is attainable without large-scale computation.

II.B.1 MAXIMUM SWIRL SPEED ESTIMATE

An upper and lower bound on the central pressure deficit achievable in a known spawning atmosphere will now be set forth by use of the hurricane model just presented, of hydrostatics, and of the thermodynamics of moist and dry air. Specifically, the weights of various columns of air in the storm will be determined in light of different moisture content and thermodynamic processes involved. The bounds on the central pressure deficit can then be translated into an estimate of bounds on the maximum swirl speed through dynamics (the radial momentum equation). Fletcher (1955) had suggested use of the cyclostrophic balance once pressure deficits were known, and Malkus (1968) had suggested that pressure deficits could be calculated from moist adiabatic considerations for the eyewall and dry adiabatic considerations for the eye. Here the concepts are combined to achieve quantitative bounds, but just as important, to demonstrate that <u>hurricane speeds could be achieved without requiring any augmenting enthalpy transfer from the ocean whatever</u>.

The first step is to neglect the frictional boundary layer II, which is relatively thin and across which, except for hydrostatic variations, the pressure does not change according to lowest-order boundary layer theory.

The variation of pressure p, density ρ, and temperature T with height above the ocean z, for any ambient tropical atmosphere in which a hurricane forms, may be computed from

$$p_a = \rho_a R_a T \qquad (a = \text{dry air}); \tag{II.B.1}$$

$$p_v = \rho_v R_a T/\sigma \qquad (v = \text{water vapor};\ \sigma = 0.622); \tag{II.B.2}$$

$$p = p_a + p_v,\ \rho = \rho_a + \rho_v,\ p_v = P(T)\ (RH); \tag{II.B.3}$$

$$\frac{dp}{dz} = -\rho g; \tag{II.B.4}$$

$$T = f(p),\ RH = g(p), \tag{II.B.5}$$

where the temperature profile f(p) and the relative humidity (RH) profile g(p) are taken as known from measurement. The saturation pressure P(T) is well-tabulated for vapor and liquid phases above freezing, and vapor and solid below freezing (Keenan and Keyes 1936); a convenient and accurate expression for P(T) in mb was given by Tetens (Murray 1967):

$$P(T) = 6.1078 \exp\left[\frac{a\,(T - 273.16)}{(T - b)}\right], \qquad \text{(II.B.6a)}$$

$$\left.\begin{array}{l} a = 21.8745584 \\ b = 7.66 \end{array}\right\} \text{over ice;} \qquad \left.\begin{array}{l} a = 17.2693882 \\ b = 35.86 \end{array}\right\} \text{over water,} \qquad \text{(II.B.6b)}$$

where T is in °K. The integration proceeds from the sea level upward in altitude z; data typically extend from about 1000 mb to 150 mb. The top of the storm is normally taken as the height at which the ambient total stagnation enthalpy (for which the kinetic energy contribution is negligible) recovers its sea-level value, as noted earlier; here, however, a slightly different procedure explained below will be used.

In a fully developed storm the air rising up the eyewall in Carrier's model follows a moist adiabat based on the <u>sea-level ambient</u> state (until late in the storm when an ambient state above sea-level should serve as the reference state for the moist adiabat, but by then the storm has weakened from the maximum intensity level of interest here). Thus for the eyewall one integrates dH = 0 where the total stagnation enthalpy H is given by

$$H = cpT + LY + gz + q^2/2 \qquad \text{(II.B.7)}$$

where q is the wind speed relative to the earth, L the latent heat of condensation per unit mass, and Y the mass fraction of water vapor (ρ_v/ρ). After manipulation with (II.B.1)-(II.B.4), dH = 0 may be written (L held constant)

$$\frac{dT}{dp} = \frac{\frac{1}{\rho} + \frac{L\sigma}{p^2 x^2}\,P - \frac{d(q^2/2)}{dp}}{c_p + \frac{L\sigma}{px^2}\frac{dP}{dT}} \qquad \text{(II.B.8)}$$

where the $[d(q^2/2)/dp]$ contribution is negligible and

$$x = 1 - \frac{(1 - \sigma)P}{p} \doteq 1 \leftrightarrow p \doteq p_a \quad . \qquad \text{(II.B.9)}$$

A slightly different mode of derivation gives a similar result:

$$\frac{dT}{dp} = \frac{\dfrac{R_a T}{p_a} + \dfrac{L\sigma}{p\, p_a} P}{c_p + \dfrac{L\sigma + R_a T}{p_a} \dfrac{dP}{dT}} \qquad \text{(II.B.10)}$$

This alternate form is used in any calculations reported here. The procedure is to use the dry adiabatic relation $T \sim p^{\gamma/(\gamma - 1)}$, $\gamma = 1.4$, from sea-level ambient conditions $[T(p_s) = T_s$, $RH(p_s) = RH_s$, where subscript s denotes sea-level ambient conditions (given)]; where RH = 1, one switches to the moist adiabat (II.B.10) and continues. The integration is terminated at that pressure p for which the temperature calculated from the moist adiabat (II.B.10) and from the ambient (II.B.1)-(II.B.5) are equal; this temperature is denoted T_1, and the height above sea-level at which T_1 occurs is denoted z_1 (the "lid" on the cyclone). One then integrates (II.B.1)-(II.B.4) and (II.B.10) from $z = z_1$ (where $T = T_1$ and $p = p_1$) to $z = 0$; RH = 1 during this integration since typically sea-level autumnal tropical air becomes saturated when raised even 20 mb. If no eye existed in the vortex -- as seems to be the case for some tornadoes and waterspouts -- then the just-calculated $p(z = 0) \equiv p_e$, $\rho(z = 0) \equiv \rho_e$ would characterize conditions at the center of the vortex.

In a mature hurricane a pressure deficit in excess of $(p_s - p_e)$ is achieved by having rain-out air entrained from the eyewall sink in a relatively dry eye under adiabatic recompression. Thus in a hurricane $(p_s - p_e)$ is a lower bound on the central pressure deficit. For an upper bound on the deficit that may be achieved, one may adopt the idealized model that the eye is completely dry (so no compressional heat is lost to re-evaporation) and that the air entrained into the eye is drawn from the top of the eyewall (or, in any case, has $T = T_1$, $p = p_1$, $Y = 0$ at $z = z_1$).

The relevant equations are (II.B.1)-(II.B.4), RH = 0, and $T \sim p^{(\gamma - 1)/\gamma}$; integration in the direction of decreasing z yields $p(z = 0) \equiv p_c (< p_e < p_s)$ and $\rho(z = 0) = \rho_c (< \rho_e < \rho_s)$ -- the density discrepancies so calculated are at most twenty-five percent and Fletcher (1955) estimates the density does not vary by even fifteen percent, so the density may be held constant at its ambient value throughout the dynamical calculations now discussed.

If one adopts the cyclostrophic balance, holds ρ constant at (say) ρ_s, and (since the core is observed not to rotate) lets

$$v(r) = \begin{cases} 0 & 0 \leq r \leq R \\ (v)_{max} \, (R/r)^n & R \leq r \leq \infty \end{cases} , \qquad \text{(II.B.11)}$$

then

$$(v)_{max} = \left[2n \left(\frac{p_s - p_c}{\rho_s}\right)\right]^{1/2} . \qquad \text{(II.B.12)}$$

First, for a one-cell vortex (when there is no eye so the moist adiabat calculation is appropriate all the way to the axis), a rigid-body-like rotation lies near the core so then

$$v(r) = \begin{cases} (v)_{max} \, (r/R) & 0 \leq r \leq R \\ (v)_{max} \, (R/r)^n & R \leq r \leq \infty \end{cases} , \qquad \text{(II.B.13)}$$

and from the cyclostrophic balance

$$(v)_{max} = \left[\frac{2n}{n + 1} \left(\frac{p_s - p_e}{\rho_s}\right)\right]^{1/2} \qquad \text{(II.B.14)}$$

Next, although power-law decays of swirl with radial distance are frequently adopted and suffice for current purposes, it will become evident that other forms are at least as plausible, and more convenient, for $r \geq R$. In any case, Miller (1967) suggested from limited data that $0.5 < n < 0.65$ usually

suffices; earlier, Byers (1944, p. 435) recommended $n \doteq 0.5$ and Hughes (1952) considered $n \doteq 0.6 - 0.7$ suitable for an average hurricane. The upshot is that estimates will be made on the limits $n = 0.5$ and $n = 1.0$. Finally, the gradient wind equation would probably be more appropriate than the cyclostrophic equation, but the more complicated formula would give maximum speeds reduced by only five percent from those obtained from the simple forms (II.B.12) and (II.B.14).

For the typical ambient West Indies tephigram for September given by Jordan (1957) (extending to the 130 mb level), one calculates a pressure deficit of 58 mb for a one-cell storm (no eye) and 137.5 mb for a two-cell storm (with an eye). These deficits translate into bounds on $(v)_{max}$ of 130.2 mph for $n = 0.5$ and 159.5 mph for $n = 1.0$ for the storm without an eye, and 224.8 mph for $n = 0.5$ and 346 mph for $n = 1.0$ for a storm with an eye. Further results are given in Figures (II.B.2)-(II.B.4).

II.B2 THE SWIRL-DIVERGENCE RELATION FOR THE FRICTIONAL BOUNDARY LAYER

Because the maximum speed achieved in a tropical cyclone is rarely much over 200 mph, as demonstrated in Section II.B.1, the Mach number rarely reaches even 0.3. Hence, when examining the dynamics (as opposed to the energetics), an incompressible constant-property model suffices.

A steady axisymmetric flow of an incompressible fluid is now studied to confirm the crucial point that, under rapid swirling, there is downflux from region I to region II, and sufficient downflux enters the surface frictional layer to account for the mass flux up the eyewall. The analysis will be carried out in a noninertial coordinate system rotating at the constant speed of that component of the rotation of the earth which is normal to the local tangent plane. Because the boundary-layer divergence under an impressed swirl (the major constraint furnished by the boundary layer on the inviscid flow above it) is relatively small in magnitude, careful formulation and solution of the coupled quasilinear parabolic partial differential equations and boundary conditions describing the layer is required.

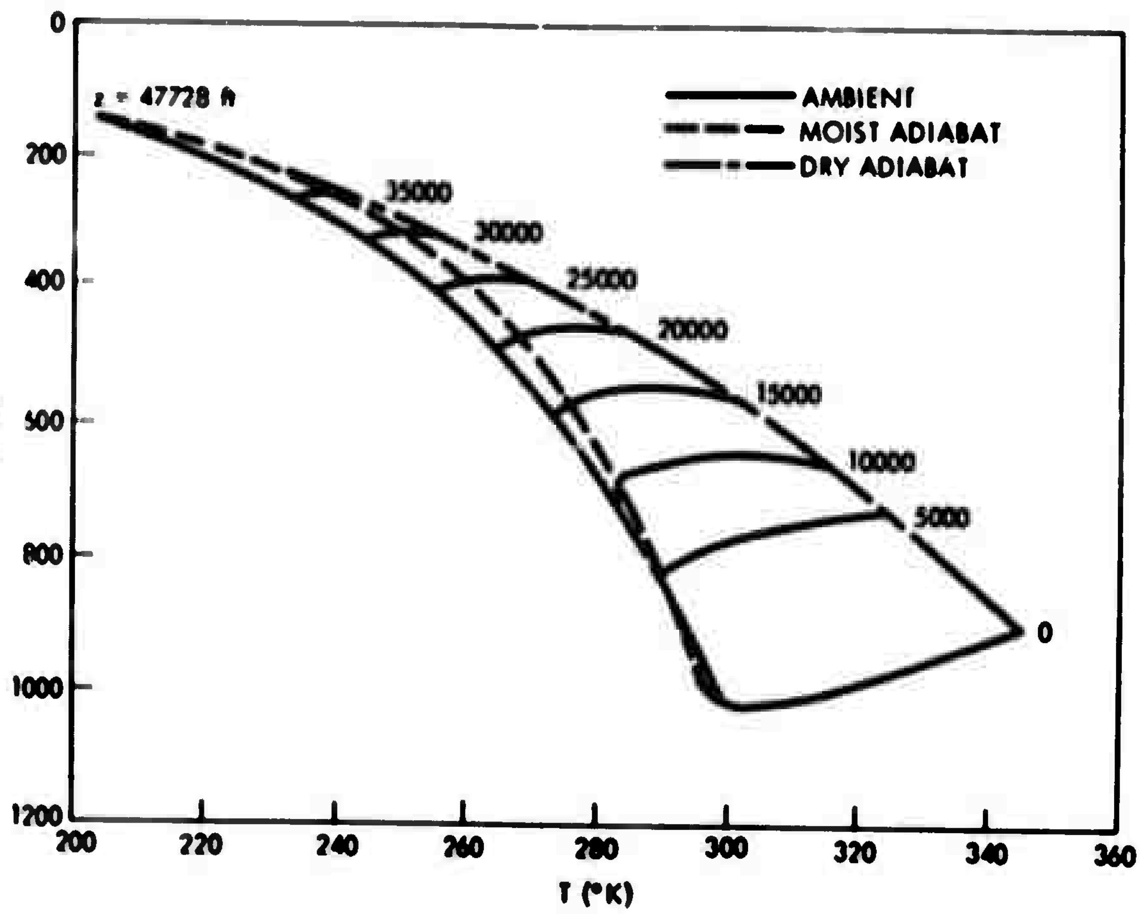


Fig. II.B.2 This tephigram gives thermodynamic loci determining upper and lower bounds on sea-level central pressure deficits in a hurricane spawned in a known ambient environment. The ambient pressure-temperature curve is based on data for the Caribbean in September by Jordan (1957). The curve labeled moist adiabat is based on having sea-level ambient air rise dry adiabatically until saturation, and thenceforth moist adiabatically. The sea-level pressure associated with such a column of gas gives a lower bound on the central pressure deficit from ambient. An upper bound on the deficit is furnished by having the air that rose on the so-called moist adiabat, recompressed dry adiabatically from the top of the storm back down to sea level. Altitudes are associated with the thermodynamic state by use of hydrostatics and the equations of state for dry air and water vapor.

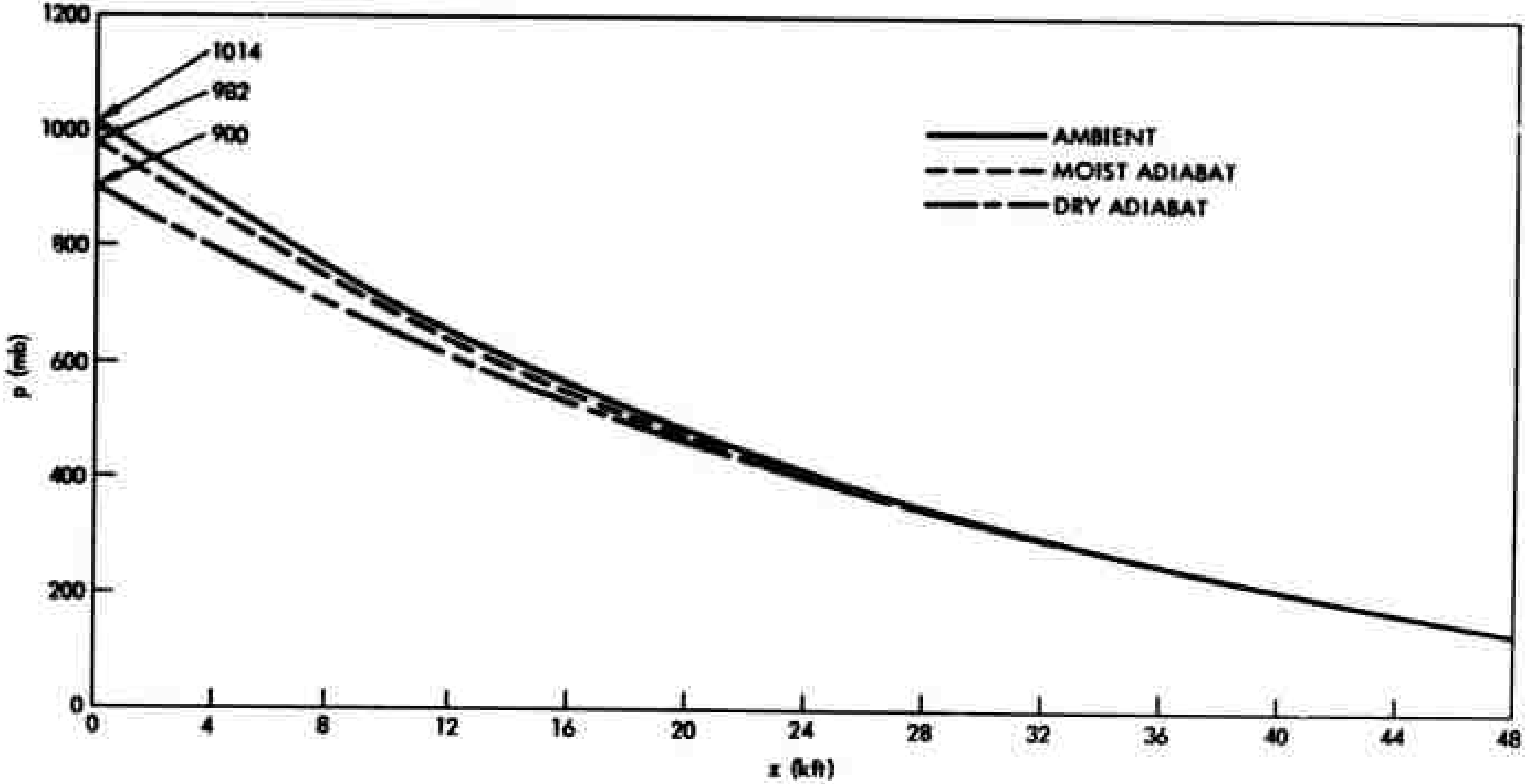


Fig. II.B.3 The pressure-altitude relation for each of the three thermodynamic loci of Fig. II.B.2 is given here more explicitly. Clearly a two-cell storm with a dry adiabatic core can support much higher swirling speeds than a one-cell storm, in which the so-called moist adiabatic ascent characterizes the flow at the axis of rotation.

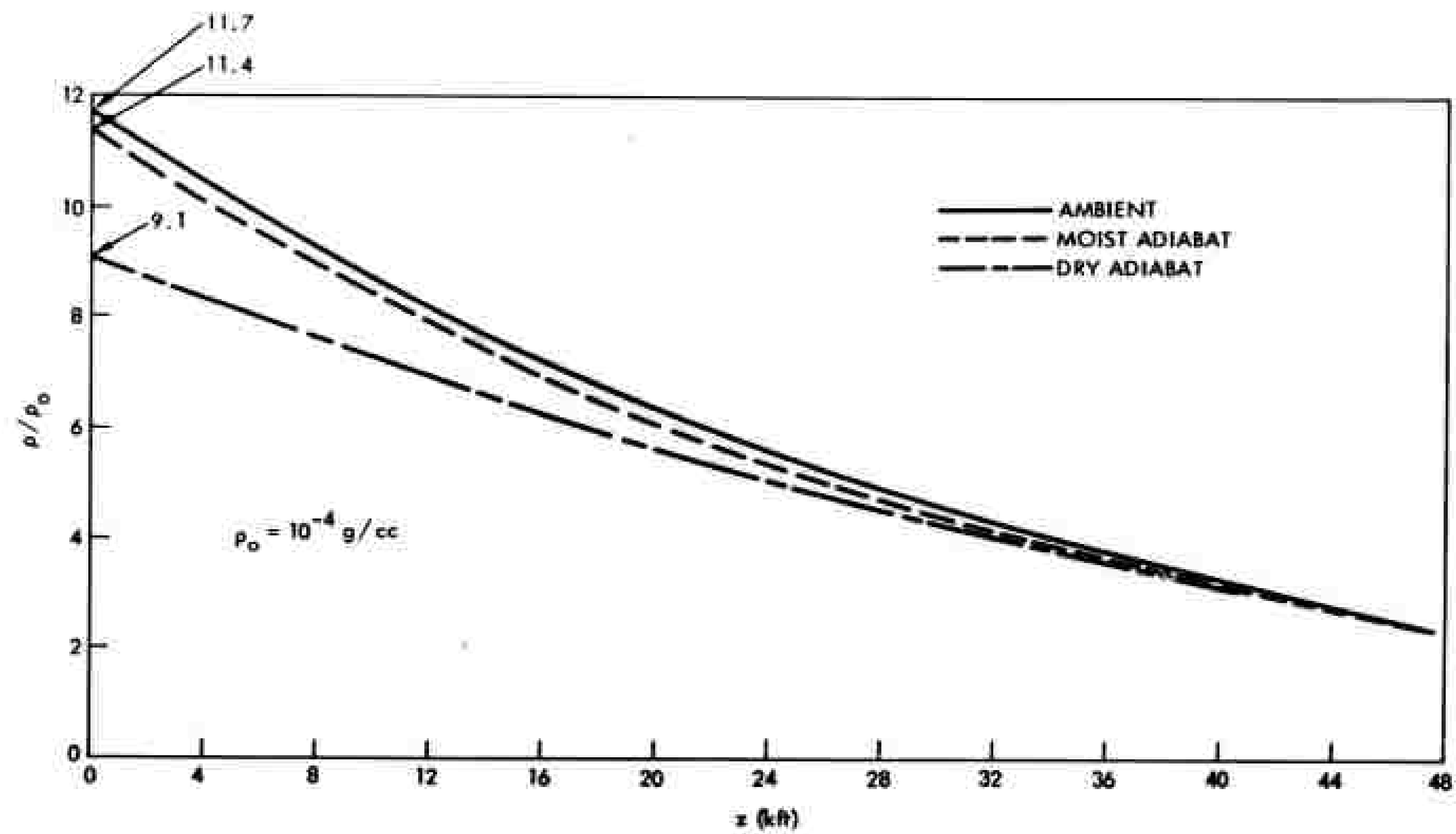


Fig. II.B.4 The density-altitude relation is given for each of the three thermodynamic loci of Fig. II.B.2. The density variations from ambient within the storm are less than thirty percent at any altitude for this flow field, and speeds are far below sonic everywhere.

The relevant equations are

$$\nabla \cdot \bar{q} = 0 \,, \tag{II.B.15}$$

$$\nabla(q^2/2) + (\nabla \times \bar{q}) \times \bar{q} + 2\bar{\Omega}_e \times \bar{q} = -\nabla\tilde{p} - \nu \nabla\times(\nabla \times \bar{q}), \tag{II.B.16}$$

where $\tilde{p} = (p/\rho) + (\bar{\Omega}_e \times \bar{r})^2/2 + gz$, the gravitational acceleration $\bar{g} = -g\hat{z}$, the velocity in noninertial coordinates $\bar{v} = \bar{\Omega}_e \times \bar{r} + \bar{q}$, the component of the rotation of the earth normal to the local tangent plane $\bar{\Omega}_e = \Omega\hat{z}$, and the kinematic viscosity (later given eddy-diffusivity values) is ν.

Nondimensionalization is effected by letting $\bar{q}' = \bar{q}/(\Psi_0\Omega)^{1/2}$, $p' = \tilde{p}/(\Psi_0\Omega)$, $\bar{r}' = \bar{r}/(\Psi_0/\Omega)^{1/2}$, and $E = \nu/\Psi_0$ where the Ekman number $E << 1$ and Ψ_0 characterizes the circulation away from the boundary (such as the maximum swirl speed times the radius at which it occurs). Dropping primes, one has

$$\nabla \cdot \bar{q} \,, \tag{II.B.17}$$

$$\nabla(q^2/2) + (\nabla\times \bar{q}) \times \bar{q} + 2\hat{z} \times \bar{q} = -\nabla p - E\, \nabla\times(\nabla \times \bar{q}). \tag{II.B.18}$$

These equations are studied in axisymmetric cylindrical polar coordinates

$$\bar{q} = u\,\hat{r} + v\,\hat{\theta} + w\,\hat{z},\ \bar{r} = r\,\hat{r} + z\,\hat{z} \ . \tag{II.B.19}$$

Away from the boundary (i.e., in region I) the following expansions are adopted:

$$p = \pi(r,z) + \ldots,\ v = V(r,z) + \ldots,\ w = E^{1/2}\, W(r,z) + \ldots,\ u = o(E^{1/2}). \tag{II.B.20}$$

Substitution of (II.B.20) in (II.B.17) and (I.B.18) gives the gradient-wind equation:

$$\pi_z = 0,\ W_z = 0,\ \pi_r = 2V + V^2/r \ . \tag{II.B.21}$$

Subscripts r and z here (and x and ζ below) denote partial differentiation. The axially invariant solution is complete when $\Psi(r)$ or $V(r)$ is specified [here $V(r)$ will be given]; $W(r)$ is found by matching the solution to (II.B.21) to the solution for the frictional layer II, and in this sense $W(r)$ is determined by the boundary-layer dynamics.

If $\zeta = z\,E^{-1/2}$ [which implies that the frictional layer is $O(E^{1/2})$ in thickness] and if near the boundary

$$u = u_b(r,\zeta) + \ldots,\; v = v_b(r,\zeta) + \ldots,\; w = E^{1/2} w_b(r,\zeta) + \ldots,\; p = p_b(r,\zeta) + \ldots, \tag{II.B.22}$$

then the axial component of the momentum conservation equation degenerates to $(\partial p_b/\partial\zeta) = 0$ in conventional fashion, so the pressure field in the boundary layer is known from (II.B.21). If

$$\psi = rv_b,\; \Psi = r\,V,\; \phi = ru_b,\; x = r^2,\; \tilde{w} = 2^{-1/2} w_b,\; \tilde{\zeta} = 2^{1/2}\,\zeta, \tag{II.B.23}$$

then in terms of dimensional quantities $\phi = ru/\Psi_0$, $\psi = rv/\Psi_0$, $\tilde{w} = w/(2\Omega\nu)^{1/2}$, $\tilde{\zeta} = z/(\nu/2\Omega)^{1/2}$, $x = \Omega r^2/\Psi_0$, the boundary layer thickness is $O(\nu/\Omega)^{1/2}$. In terms of quantities introduced in (II.B.23), one has from (II.B.17) and (II.B.18)

$$\phi_x + w_\zeta = 0\,, \tag{II.B.24}$$

$$2\phi\phi_x + w\phi_\zeta + (\Psi^2 - \psi^2 - \phi^2)/2x - (\psi - \Psi) - \phi_{\zeta\zeta} = 0, \tag{II.B.25}$$

$$2\phi\psi_x + w\psi_\zeta + \phi - \psi_{\zeta\zeta} = 0\,. \tag{II.B.26}$$

Matching of expansions gives

$$\zeta \to \infty:\quad \phi \to 0,\; \psi \to \Psi(x) \text{ given}, \tag{II.B.27}$$

and at $\zeta = 0$ no-slip conditions are adopted:

$$\zeta = 0:\quad \phi = w = \psi = 0\,. \tag{II.B.28}$$

Initial conditions are conveniently given by noting that at $x = x_0$, for x_0 large enough, solution is given by discarding all the nonlinear terms and retaining the linear equations treated by Ekman, in which x enters parametrically only. The solution to the balance of Coriolis, pressure, and friction forces is well known:

$$\phi = -[\nu(x)] \sin (2^{-1/2}\zeta) \exp (-2^{-1/2}\zeta), \qquad \text{(II.B.29)}$$

$$\psi = [\nu(x)] [1 - \cos (2^{-1/2}\zeta) \exp (-2^{-1/2}\zeta)] \, , \qquad \text{(II.B.30)}$$

$$w = 2^{-1/2} [\nu_x(x)] \left\{1 - [\sin(2^{-1/2}\zeta) + \cos(2^{-1/2}\zeta)] [\exp(-2^{-1/2}\zeta)]\right\} . \qquad \text{(II.B.31)}$$

Specifically what is sought is $w(r,\zeta\to\infty) = W(r)$ for $\nu(r)$ of interest. For r large, from (II.B.31)

$$w(r,\zeta\to\infty) = W(r) = 2^{-1/2} \nu_x(x) \, . \qquad \text{(II.B.32)}$$

Numerical integration by finite-difference methods is formidable because the flow component in the time-like direction u is, in successively thinner strips lying parallel to the boundary, alternately in the direction of integration (stable) and opposite to it (unstable). Though the radial flow is, on net, in the direction of integration, the integration is marginally stable and no numerical results <u>of real use</u> in the <u>hurricane</u> problem are known to the authors.

At Carrier's suggestion, George (Carrier, Hammond, and George 1971) and Dergarabedian and Fendell (1971) independently but simultaneously applied the method of weighted residuals to the boundary value problem, and found that, except near the eyewall where the method was inadequate, the linear result given in (II.B.32) sufficed for the nonlinear problem as well. Thus, for a form like $\nu = A(1 - x/x_0)$, for which $\phi(r_0,\zeta) = 0$, $w(r,\zeta\to\infty) \to -(A/x_0)$, a small positive constant (equivalent to about 0.005 mph downdraft for physically interesting values).

The reason weighted-residual calculations fail near the axis, as discovered by Carrier (1971a) by modified Oseen linearization and by Burggraf, Stewartson, and Belcher (1971) by semi-numerical analysis, is that the structure of the boundary layer so varies with radial distance that adequate representation in terms of one set of orthonormal polynomials is difficult. Far from the axis of symmetry, friction is important across the entire layer of thickness $O(\nu/\Omega)^{1/2}$; near the axis, friction is significant only is a small sublayer near the wall of thickness $O(r^2\nu/\Psi)^{1/2}$, and the remainder of the inflow layer of $O(\nu/\Omega)^{1/2}$ thickness is inviscidly controlled. Still, for conditions of interest in hurricanes, (II.B.32) is everywhere correct to within a factor of two, and often far better. Some results are given in Figure (II.B.5).

II.B3 THE ENERGETICS OF THE FRICTIONAL BOUNDARY LAYER AND THROUGHPUT SUPPLY

For the frictional boundary layer II and throughput supply I, one takes the following approximations as adequate for the quasisteady mature phase:

1. The Prandtl and Schmidt numbers are unity;
2. the hydrostatic approximation holds;
3. the boundary layer approximation holds (derivatives normal to the boundary exceed those tangential to the boundary, but velocity components parallel to the boundary exceed those normal to the boundary);
4. the eddy transfer is adequately modeled by the laminar flux-gradient relations for diffusion of mass, momentum, and heat (as given by Fick, Newton, and Fourier, respectively), except that the augmented kinematic (eddy) viscosity may vary with radial position (but not with axial position); and
5. the mixture of dry air and water vapor may be taken as a perfect gas with constant heat capacity over the range of temperatures of interest here.

In view of the limited understanding of quantitative formulation of cumulus convection and turbulent transfer, on the cyclone scale of interest here these five approximations seem reasonable. It can then be shown that

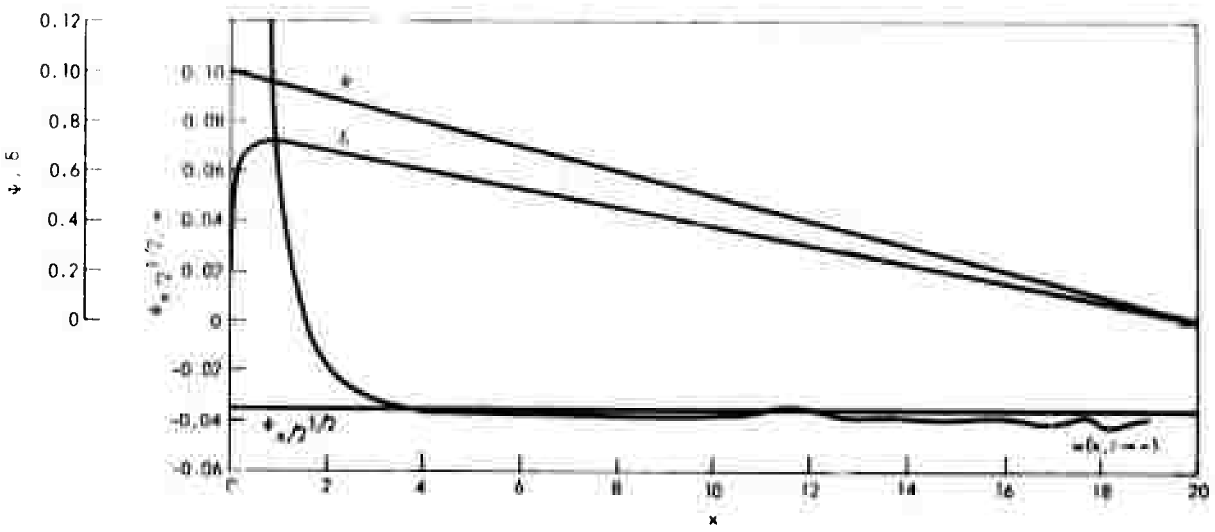


Fig. II.B.5a Nondimensional results for the frictional boundary layer obtained by the method of weighted residuals are presented [from Dergarabedian and Fendell 1971)]. The divergence $w(x, \zeta \to \infty)$ and the volumetric flux $\delta = -\int_0^\infty \phi(x, \zeta)\, d\zeta$ are presented for the impressed swirl $\Psi = 1 - x/x_1$, $x_1 \doteq 20$, believed pertinent to a hurricane outside the eyewall. Except near the axis where nonlinear inertial effects dominate, the linear Ekman layer result, $w(x, \zeta \to \infty) = \Psi_x/2^{1/2}$, is an excellent approximation to the numerical results. The volumetric flux $\delta(x)$ is thus linearly proportional to $(x_1 - x)$ to good approximation. Normalized residuals indicate large errors for $x < 3$, and discount the premature eruption as an artifact of the method, as explained by Carrier (1971a). An improved solution for small x indicates the adequacy of the linear Ekman solution to within a factor of two. Since $\nu \doteq (1/75)\ \text{mi}^2/\text{hr}$ and $\Omega = (1/16)$ rad/hr, dimensionally the results imply the boundary layer is of thickness $O(\nu/\Omega)^{1/2} = O(1\ \text{mi})$, the downflux into the boundary layer is $(2\nu\Omega)^{1/2}\, w(x, \zeta \to \infty) = O(5 \times 10^{-3}\ \text{mph})$, and the volumetric flux erupting up the eyewall is $(2\pi^2\, \Psi_0^2\, \nu/\Omega)^{1/2}\, \delta(x) = O(7 \times 10^3\ \text{mi}^3/\text{hr})$ where Ψ_0 characterizes the eyewall angular momentum per unit mass. The implication is that the fluid initially in the boundary layer sustains the hurricane for about a week and the fluid stored above the boundary layer (with supplementary replinishment of moisture from the ocean) can readily sustain the hurricane for more than a week more.

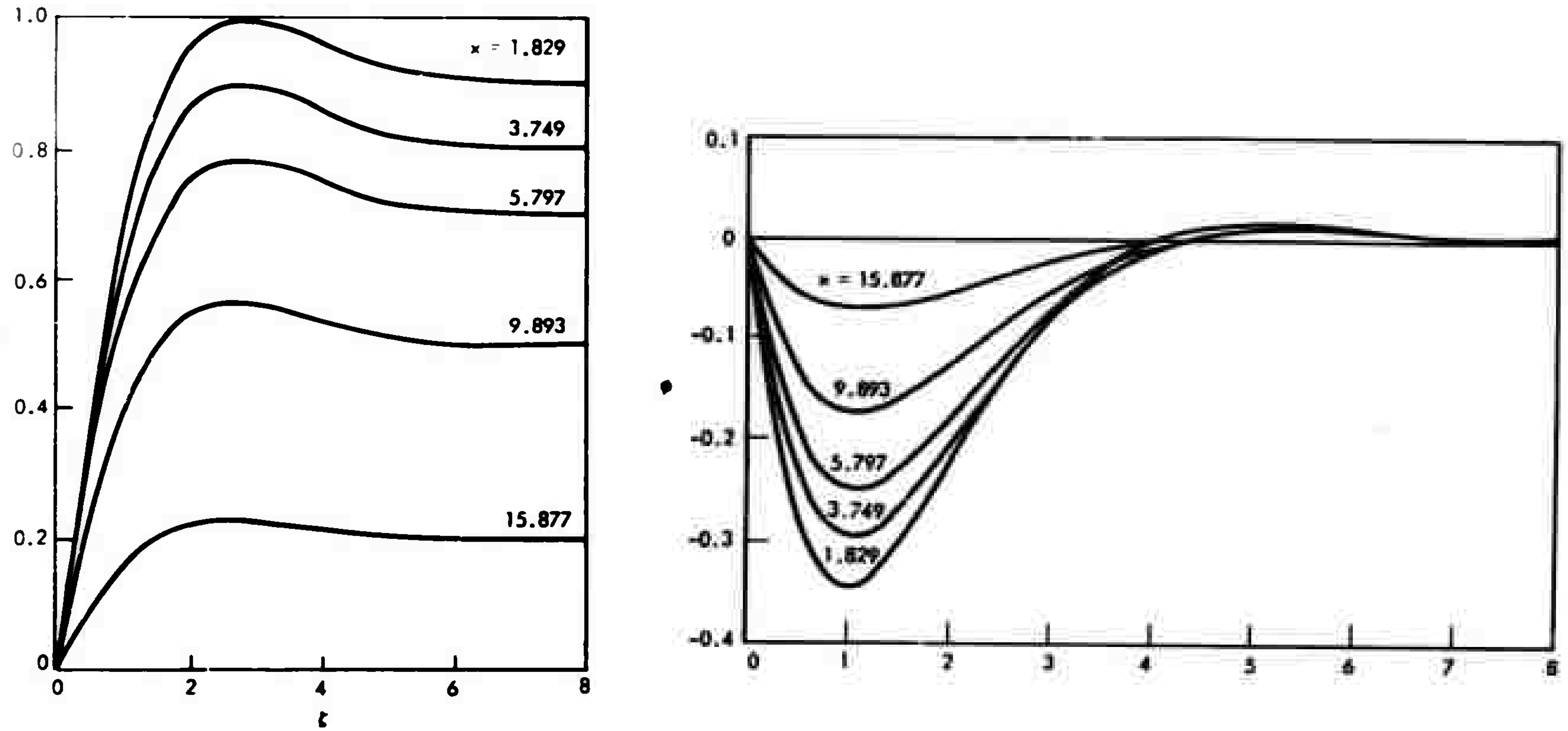


Fig. II.B.5b The axial profiles of the modified radial and azimuthal velocity components, ϕ and ψ respectively, for several radial positions x, for the impressed swirl given in Fig. II.B.5a. Again, these weighted-residual results are erroneous at $x < 3$, but for larger x correctly indicate both the reversing radial-flow profiles (with net flux inward) and also the swirl overshooting its outer asymptotes. The maximum magnitude of the radial component is about one-third that of the impressed swirl at a typical radial position.

the following equation, a generalization of ones given by Crocco in fluid dynamics and Shvab and Zel'dovich in combustion, holds in the meteorological context of interest here:

$$u \frac{\partial H}{\partial r} + w \frac{\partial H}{\partial z} = \nu \frac{\partial^2 H}{\partial z^2} + R , \qquad \text{(II.B.33)}$$

where R denotes radiational loss, taken by Carrier, Hammond and George to be representable as

$$R = - f(z)\, H ,$$

with $f(z)$ (to be discussed below) known. The definition of H has been given in (II.B.7). The boundary-initial conditions are taken to be

$$z = 0: \ H = H_s ; \ z = z_1 : \ H = H_1 ; \qquad \text{(II.B.34a)}$$

$$r = r_o : \ H = H_a(z) , \qquad \text{(II.B.34b)}$$

where, again, r_o is the outer edge of the storm and z_1 is the top of the storm. The quantities $H_a(z)$, $H_1 = H_a(z_1) = H_a(z = 0)$, and $H_s = H_a(z = 0)$ are all given. Specifically $H_a(z)$ is the <u>ambient</u> distribution of, effectively, the equivalent potential temperature times c_p. Taking $H_s = H_a(z = 0)$ implies that for $r > R$ (where R is the radius of the eyewall), the ocean surface temperature is a constant and the water vapor mass fraction (which takes on its saturation value at the nominally plane sea surface $z = 0$) is virtually independent of pressure. Since the thermal conductivity of water greatly exceeds that of air, uniform sea-surface temperature appears to be a good approximation. That $H_1 = H_a(z = 0)$ follows from the definition of the lid on the storm.

The terms, from left to right, in (II.B.33) represent radial advection; axial convection; turbulent diffusion and cumulus convection (both parameterized in ν, which will henceforth be treated as constant, though this is not necessary); and radiation loss. Throughout I, and in II at $r = r_o$

since $v(x_0) = 0$ by choice [cf. (II.B.29)] and since $w(\partial H/\partial z)$ can be neglected as explained below,

$$\nu \frac{\partial^2 H}{\partial z^2} = f(z)\, H \; ; \qquad \text{(II.B.35)}$$

i.e., $f(z)$ is chosen to permit $H(z) = H_a(z)$, given. Carrier, Hammond, and George (1971) at this point invoke Oseen linearization arguments. First, they take radial advection as uniformly negligible next to axial convection so the solution is only parametrically dependent on r. This implies slow change of H with r, and is confirmed a posteriori by substitution of the solution into (II.B.33)-(II.B.34). In fact, if one takes $H(r,z) = H_a(z)$, which obeys the boundary and initial conditions, and substitutes this into the equation, one finds $u(\partial H/\partial r) = 0$; for a typical value of z

$$\frac{w(\partial H_a/\partial z)}{\nu(\partial^2 H_a/\partial z^2)} \leq O(10^{-1}) \; ; \qquad \text{(II.B.36)}$$

and

$$\nu \frac{\partial^2 H_a}{\partial z^2} = f(z)\, H_a \qquad \text{(II.B.37)}$$

by definition. The function $w(r,z)$ is available from results from Section II.B.1, which indicate that $(w)_{max} = w(r,\zeta\to\infty) = W(r)$, the value at the outer-edge of the boundary layer. In the eyewall w is increased by simple continuity considerations, to two orders of magnitude larger than its maximum value in I or II at $r > R$. Hence, in the eyewall, vertical convection dominates,

$$w \frac{\partial H}{\partial z} = 0 \; , \qquad \text{(II.B.38)}$$

and the relevant boundary data for this hyperbolic suboperator is that given at $z = 0$ in (II.B.34a).

All this discussion, much more carefully argued in Carrier, Hammond, and George (1971), leads to two simply stated but exceedingly important results:

$$r_0 \geq r > R: \quad H(r,z) \doteq H(r_0,z) \equiv H_a(z) \; ; \qquad \text{(II.B.39)}$$

$$R \geq r > r_e: \quad H(r,z) \doteq H_a(z = 0) \; ; \qquad \text{(II.B.40)}$$

where r_e is the outer radial extent of the eye. These last two equations state that for the mature hurricane:

1. throughout the frictional boundary layer and the throughout supply, outside the eyewall, to within a ten percent error, the total stagnation enthalpy is fixed at its ambient stratification. Furthermore, the ten-percent correction is readily seen to be a decrease of H with z such that the enthalpy gradient at z = 0 is increased slightly.
2. in the eyewall, the total stagnation enthalpy is constant at its sea-level value, i.e., the air is rising on a moist adiabat.

Numerical values of interest are the ambient net sea/air enthalpy transfer and the eddy viscosity (Carrier, Hammond, and George 1971):

$$-\rho\nu(\partial H/\partial z)|_{z=0} \doteq -\rho\nu(\partial H_a/\partial z)|_{z=0} = 1.8 \times 10^5 \text{ ergs/cm}^2\text{sec} \; ; \qquad \text{(II.B.41)}$$

$$\nu = 2.7 \times 10^5 \text{ cm}^2/\text{sec} \qquad \text{(II.B.42)}$$

II.C THE RIEHL-MALKUS MODEL

The alternate theory of hurricane maintenance is that "... it is postulated that lowering of surface pressures in hurricanes arises mainly through an 'extra' oceanic heat source in the storm's interior" (Malkus and Riehl 1960, p. 12). To understand why such an 'extra' oceanic heat source must be postulated by Riehl and Malkus, one must reconstruct their logic.

First, the existence of a frictional inflow boundary layer is acknowledged: "The inflow into a hurricane is confined mainly to low

levels. Subcloud air is accelerated inward along spiral-shaped trajectorie ; acceleration results from excess work done by pressure gradients over frictional retardation" (Malkus and Riehl 1960, p. 3). "... The shearing stress vanishes at the top of the inflow layer in accord with the hypothesis that $\partial v/\partial z$ is very weak above the ground layer" (Ibid.,p. 4). In a typical hurricane the boundary layer is taken as 1.1 km thick, with the eye boundary at 25 km and the extent of storm about 500 km in the Atlantic and 800-1000 km in the Pacific. Malkus and Riehl choose to let the relative vorticity, rather than relative velocity, vanish at the outer edge -- the result is an open system with radial inflow at the outer edge, as opposed to the closed system preferred by Carrier and his co-workers. In a sample moderate hurricane Malkus and Riehl calculate an <u>efflux out of the boundary layer</u> from the outer edge of the eye out to 500 km, an efflux that increases with decreasing radius: "an average ascent rate of about 30 cm/sec or 1 km/hr is required at the top of the inflow layer" (Ibid., p. 7). This result supposedly holds even though the impressed swirl decreases with increasing radius; hence the result contradicts both linear and nonlinear theories for the surface frictional layer under a swirling flow. There is no demonstration of internally consistent dynamics for the open model of Malkus and Riehl. The point of controversy is that the flux through the boundary layer does not sink down into the boundary layer (as in the Carrier model), but rather flows radially inward from the outer edge.

The boundary layer air, according to Malkus and Riehl, undergoes adiabatic expansion as it spirals inward toward the center, yet it remains isothermal. This requires a vast, rather localized increase in sea-to-air transfer between the ocean and the contiguous atmosphere (in contrast to the more spread out, unaugmented transfer pictured in Carrier's theory). That the gradient normal to the air/sea interface of temperature and of water vapor mass fraction is large enough to be consistent with vastly increased air/sea transfer is an article of faith:

> In the outskirts of a hurricane the temperature of the inflowing air drops slowly due to adiabatic expansion during (horizontal) motion toward lower pressure. It is one of the remarkable observations in hurricanes that this drop ceases at pressures of 990-1000 mb and that thereafter isothermal expansion takes place. Presumably, the temperature difference between sea and air attains a value large enough for the oceanic heat supply to take place at a sufficient rate to keep the temperature difference constant. (Malkus and Riehl 1960, p. 9).

The actual transports [between sea and air], of course, are very large in the hurricane compared to the trades. Sensible heat pickup is 720 cal/cm^2/day, and increase by a factor of 50 over the trades..; latent heat pickup is 2420 cal/cm^2/day, higher by a factor of 12-13. (Malkus and Riehl 1960, p. 12).

Thus to achieve an extreme storm in the framework of this model, [turbulent] transfer coefficients enhanced by a factor of 3-4 appear to be necessary. (Malkus and Riehl 1960, p. 16).*

The Riehl-Malkus theory that greatly augmented heat and mass transfer sustains the tropical cyclone has, in fact, been parameterized into all existing computer simulations [which is why all these simulations are here lumped into one category]. For example, it is interesting to note how closely the author of a well-known computer simulation (Rosenthal 1971b)

*The same arguments are made at more length elsewhere by Riehl (1954, pp. 286-287):

Many published records, notably those by Deppermann.., have proved that the surface temperature outside the eye is constant or decreases very slightly toward the center. The implications of this remarkable fact passed without notice until Byers.. drew attention to it. The temperature of the surface air spiraling toward a center should decrease if adiabatic expansion occurred during pressure reduction. For instance, air entering the circulation with the average properties of the mean tropical atmosphere should reach the 930 mb isobar with a temperature of 20.5°C and specific humidity of 17g/kg. Because of condensation, a dense fog should prevail at the ground inward from the 970 mb isobar. But this is never observed. It follows that the potential temperature of the surface air increases along the inward trajectories. We also know that the specific humidity increases and that the cloud bases remain between a few hundred and 1000 feet.

The surface air thus acquires both latent and sensible heat during its travel toward lower pressure ...

A source for the heat and moisture increment is obvious. The ocean is greatly agitated, and large amounts of water are thrown into the air in the form of spray. It is hard to say where the ocean ends and where the atmosphere begins! As the air moves toward lower pressure and begins to expand adiabatically, the temperature difference between ocean and air suddenly increases. Since the surface of contact between air and water increases to many times the horizontal area of the storm, rapid transfer of sensible and latent heat from ocean to air is made possible. In the outskirts, say beyond the 990 mb isobar, the turmoil is less and the process of heat transfer is not operative. (Riehl 1954, pp. 286-287).

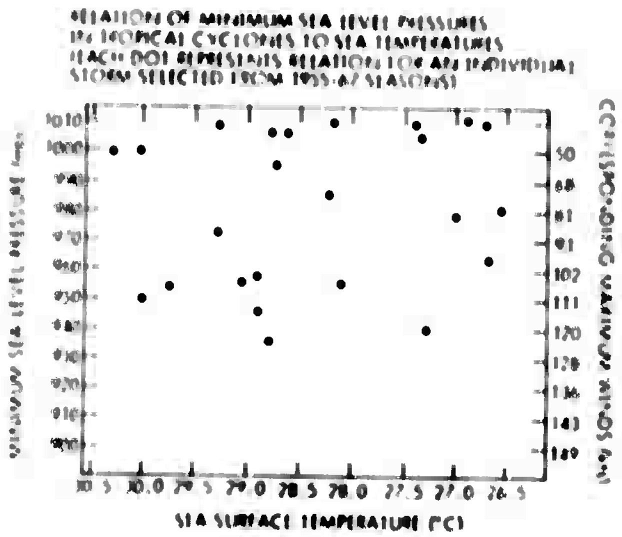
reflects the Riehl-Malkus theory in his most recent publication and how he notes similar logic in the work of another computer modeler of tropical cyclones (Ooyama 1969):

> Air-sea exchanges of sensible and latent heat have long been considered important ingredients in the development and maintenance of tropical storms. Palmén (1948) showed, on a climatological basis, that tropical storms form primarily over warm ocean waters ($T_{sea} > 26°C$). Malkus and Riehl (1960) showed that the deep central pressures associated with hurricanes could not be explained hydrostatically unless the equivalent potential temperature, θ_e, in the boundary layer was 10° to 15°K greater than that of the mean tropical atmosphere. Byers (1944) pointed out that the observed near-isothermal conditions for inward spiraling air in the hurricane boundary layer required a source of sensible heat to compensate for the cooling due to adiabatic expansion...
>
> Ooyama (1969) found drastic reductions in the strength of his model storm when the air-sea exchanges of sensible and latent heat were suppressed. He pointed out that at sufficiently large radii, the boundary layer is divergent (the so-called Ekman layer "sucking")... This subsidence tends to decrease the boundary layer θ_e since $\partial\theta_e/\partial z < 0$ in the lower troposphere. Ooyama argued that unless the energy supply from the ocean can again raise the θ_e of the boundary layer air to sufficiently large values before the inflowing air reaches the inner region, and the convective activity will diminish in those regions and, hence, the storm will begin to weaken.
>
> Ooyama's line of reasoning can be extended to show that evaporation is far more important than sensible heat flux. The air sucked into the boundary layer has a higher potential temperature than the original boundary layer air. The subsiding air has a smaller θ_e only because it is relatively dry. (Rosenthal 1971b, p. 772).

An extensive presentation has been given to the Riehl-Malkus theory, and an extensive criticism will now follow, because the theory has such wide acceptance. For example, a recent National Science Foundation document, in reviewing the work of Ooyama at NYU -- work emphasizing the reliance on high sea temperature and attendant large enthalpy transfer from sea to air --, states: "Besides the fact that the model has succeeded in simulating many important aspects of the hurricane, it has also demonstrated the importance of oceanic latent-heat supply to both the development and maintenance of the tropical storm" (NSF 1969, p. 96). Battan (1969, p. 117) cites Ooyama's results with no reservations: "Ooyama's research shows that

storm dynamics depends mostly on the temperature of the ocean water under the hurricane." In fact, the very title of a recent Russian article citing Riehl and Malkus papers suggests widespread acceptance: "The Power of a Tropical Cyclone as a Function of the Underlying Sea Surface Temperature" (Shuleykin 1971). The fact is that ocean temperatures are difficult to determine from currently available records, and that hurricanes cause upwelling of lower, colder water to confuse matters further (Perlroth 1967). Gentry (1969, p. 406) presents data relating "... the maximum intensity of several tropical cyclones to the temperatures of the sea beneath them and shows that both severe and weak tropical cyclones occur when ocean temperatures are relatively high. This suggests that variations in parameters other than the transfer of heat from the ocean to the atmosphere also influence the storm's intensity...". Actually Gentry's data [Figure (II.B.6)] indicate that many intense tropical cyclones lie over relatively cold ocean water (< 28.0°C). Brand (1971) cites a supposed correlation of central pressure deficit with sea-surface temperature throughout the lifespan of Hurricane Esther of 9-26 September 1961. But during one twelve-hour period of constant sea temperature, the central pressure rose 15 mb; and for three days while the sea temperature hovered about 84°F, the central pressure nonmonotonically rose from 930 to 955 mb; and when the sea temperature was at 86°F, at various times the central pressure was as low as 927 mb and as high as 953 mb. Perlroth (1967) shows that for Hurricane Ginny of 1963, for five days while the sea temperature hovered near 80°F, the central pressure nonmonotonically fell from 995 to 970 mb [Figure II.B.6]. While some correlation of central pressure deficit and sea surface temperature definitely exists, tropical cyclone intensity cannot be completely correlated in so simple a fashion. Thus, because of the fact that it is a starting point for almost all subsequent theoretical research on tropical cyclones, there seems reason to scrutinize the Riehl-Malkus theory carefully.

There would seem to be several errors in the Malkus-Riehl reasoning. First, even for the linearized Ekman layer at the outer edge of the tropical cyclone, it is inappropriate to use the adiabatic-expansion relation $p \sim T^{\gamma/(\gamma - 1)}$, which implies for steady motion that the entropy is constant



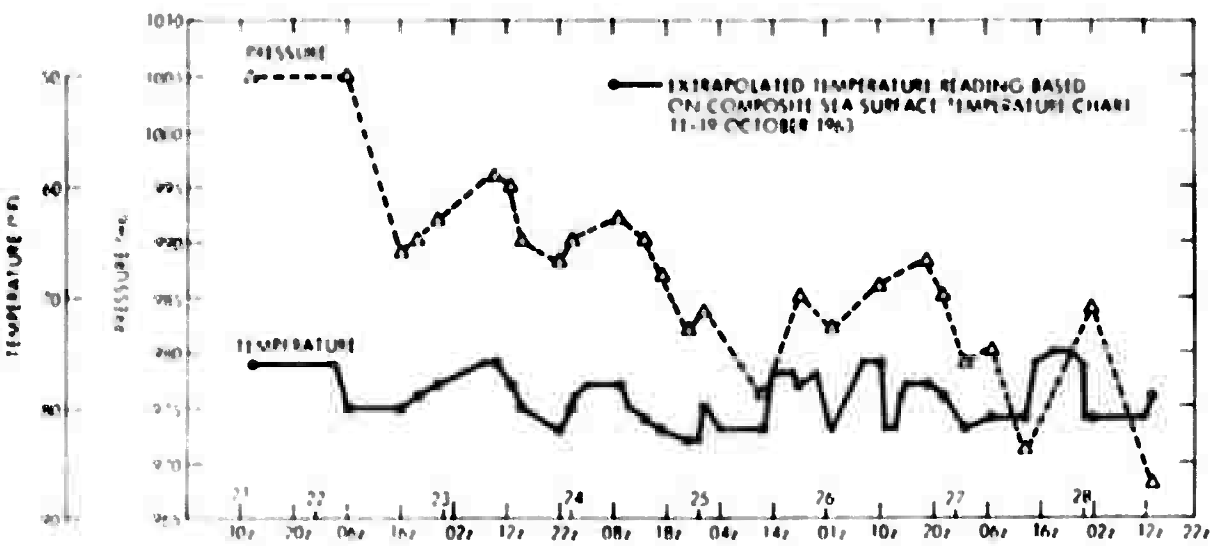


Fig. II.B.6 Above, the minimum sea-level pressure measured for a tropical cyclone is plotted against the local sea temperature [from Gentry (1969a, p. 406)]. Below, the central sea-level pressure is plotted as a function of the local sea surface temperature for hurricane Ginny, 21-28 October 1963, which passed over the Gulf Stream on 24 October [from Perlroth (1967, p. 266)].

along each streamline. Riehl (1954, p. 286) correctly asserts that neglect of diffusion gives about a 6°C drop (with condensation) and a 7.3°C drop (dry) from the mean autumnal sea-level ambient reported by Jordan (1957) for the West Indies area, in dropping from 1014 mb to 930 mb. The isentropic assumption is unjustified and the temperature decrease thereby anticipated is too large.

What temperature decrease is to be expected? It has been discussed earlier in Section II.B.3 that an approximate integral for the surface frictional layer, more than adequate for most purposes, is given by

$$H(r,z) \equiv c_p\, T + g\, z + LY + (q^2/2) \doteq H(r_o,z); \qquad \text{(II.C.1)}$$

i.e., the total stagnation enthalpy H throughout the boundary layer between the eyewall and the outer edge, is approximately fixed at the stratification of the ambient environment in which the tropical cyclone was generated. Again, r_o denotes the outer edge of the storm. At $z \simeq 0$, Y is close to its saturated value and this is primarily a function of the sea surface temperature, which is adequately modeled as a constant. Hence,

$$h_o\,(r,0^+) \equiv c_p\, T + q^2/2 = h_o\,(r_o,0^+) = \text{const.}$$

If $q \simeq 0$, $T = 299.4$ at $r = r_o$, $z = 0^+$, then at $q = 200$ mph, $T = 295.4$°K; this is a decrement of 4.0°C.

It will now be suggested that this is the decrement Riehl should have anticipated. The cyclostrophic balance [Fletcher (1955) formula] gives

$$(v)_{max} = \left[2n\left(\frac{p_s - p_c}{\rho_s}\right)\right]^{1/2}, \qquad \text{(II.C.2)}$$

if the radial profile of the swirl above the boundary layer is given by (II.B.11). Here $n = 0.6$ and $\rho_s = 1.2 \times 10^{-3}$ g/cm^3 are adopted. The ambient sea-level pressure p_s is taken as 1014 mb according to Jordan, and Riehl for his calculations adopted a case in which the central sea-level pressure fell to 930 mb. Substitution in (II.B.12) gives $(v)_{max} \cong$ 209 mph, about as large as any value reliably reported for a tropical cyclone.

Thus, for the pressure difference adopted by Riehl, equivalent to $(v)_{max}$ = 209 mph, the appropriate generalized Crocco-Shvab-Zel'dovich-like integral gives a temperature drop of about 4.3°C; Riehl's use of the adiabatic relation led him to expect a temperature drop at least three-halves as large*.

Riehl (1954), Malkus and Riehl (1960), and Rosenthal (1971b) all cite measurements that purportedly show that the sea-level atmospheric temperature in a mature tropical cyclone is constant. "This remarkable fact..." (Riehl 1954, p. 286) is based on measurements cited by Riehl, measurements made over twenty-five years ago when hurricane speeds were not commonly believed to attain 200 mph levels, and when the distinction between static enthalpy c_p T and stagnation enthalpy (c_p T + $q^2/2$) [normally unimportant in meteorology but often significant in modern high-speed aerodynamics] was not always carefully observed. The fact is that accurate sea-level measurements in intense hurricanes are not a simple matter to this day because of reliance on make-shift combinations of obsolete military aircraft and radar (Meyer 1971). The belief here is that the static temperature actually decreases as predicted by the Carrier theory when properly measured.†

*It is interesting to note that according to Riehl (1954, p. 286) a dry-adiabatic expansion from 1014 to 970 mb would cause condensation. Equivalently, he is stating that about a 3.7°C drop would cause condensation in sea-level tropical air in the hurricane season. Since even an intense (200 mph) hurricane really gives a total drop of only 4.0°C, one sees why the boundary layer is quite cloud-free in as far as the eyewall.

†Palmén and Newton (1969, p. 478) still cite, second-hand, measurements over thirty-five years old asserting the constancy of the temperature in the surface frictional layer. One must recall that aircraft penetration of hurricanes is less than twenty years old. They also cite a 1954 work by Arakawa in which the wet-bulb temperature held nearly constant during the passage of an 898-mb typhoon over a Japanese naval fleet; this is interpreted to prove that the equivalent potential temperature of the surface air rose 25°K in spiraling in from the outskirts to the center, and hence that an internal heat enthalpy source in the form of augmented sea-air latent and sensible heat transfer is operative. However, reference to even a recent book on meteorology [e.g., Hess (1959, p. 61)] will show that the dynamic contribution $q^2/2$ to the total enthalpy of the gas is neglected in computing the temperature from the measured wet-bulb temperature; this normally insignificant correction makes only a little over a one-percent contribution to the total enthalpy even in a hurricane. But confusion about this small effect has had a profound influence on the evolution of tropical cyclone theory. Actually, radially constant wet-bulb temperature confirms (II.C.1).

Thus one is led to the conclusion that Riehl believed (1) the hurricane draws upon local ambient air (not warm moist air sinking into the boundary layer); (2) a large thermal fall occurs while spiraling through the boundary layer (at least 50% above what seems correct); and (3) the isothermal flow, purportedly measured in the boundary layer, occurs (whereas it almost surely does not). The result is that Riehl decided that an internal oceanic heating source was necessary and postulated greatly augmented heat and mass transfer from ocean to atmosphere in a hurricane, especially as the center is approached (whereas the result compatible with the conservation laws is that the increase in latent and sensible heat transfer where a hurricane lies over what would be transferred in its absence is at most five or ten percent, and this slight augmentation occurs at the outer edge and the slight augmentation falls off as the center is approached). This basic slip is incorporated into every numerical model known to the authors and raises questions about many conclusions drawn from the extensive programming, computer solution, and interpretation that has gone on for several years now.*

Rosenthal (1971b), in response to publications of the authors, now concurs that greatly augmented sensible heat transfer is dispensible, but remains resolute that greatly augmented latent heat transfer from the ocean to the atmosphere -- critical to his theory and to Ooyama's -- is necessary for hurricane generation and maintenance. His statements that higher-level

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*The assertion by Riehl (1954, p. 287) that much spray is tossed into the air within hurricanes does not in itself assure augmented net enthalpy transfer from sea to air, since heat must be drawn from the air to evaporate the drops for later condensation of the water vapor in the eyewall and inner rainbands. Incidentally, one must also be careful about extrapolating empirical laws relating ocean evaporation to wind speed to near-saturation, high-speed conditions for which they were not devised. Finally, the greatly augmented latent and sensible heat sea-to-air transfer postulated by Riehl and Malkus (1960, p. 17) amounts to but about three percent of the total enthalpy of the inflowing tropical air. Yet, even acknowledging this, they insisted that this marginal amount is critical to both genesis and maintenance of hurricanes. The Carrier model states that the increase postulated by Riehl and Malkus is an order of magnitude too large and is probably not <u>the</u> important critical factor.

air sinking into the boundary layer is relatively dry and of lower equivalent potential temperature may arise from misunderstanding about the quasisteady nature of the Carrier model and supporting analysis for the mature hurricane. The air slowly sinking into the boundary layer at about 0.005 mph is enriched by plume and radiative transfer of water vapor and heat from the ocean to atmosphere. These ambient mechanisms, as has been repeatedly emphasized, persist, neither augmented nor decremented to any important degree, within the hurricane; they help to compensate for rain-out in the outer spiral bands. If indeed the ambient tropical profile for equivalent potential temperature does persist with little change in the hurricane, much of the sea/air transfer continues to pass across the boundary layer with little dimunition, just as in the ambient; the enthalpy enriches the air in the 700-900 mb strata such that by the time this air sinks into the boundary layer, it is much like the air originally in the 900-1000 mb strata. Only as the hurricane leaves the tropical oceans is this normal transfer reduced; the air entering the boundary layer eventually is of lower equivalent potential temperature since it comes from air which is originally higher in the tropical ambient (hence colder and drier) and which is not appreciably enriched as it descends. In this way traverse over ocean patches of varying temperature can cause the well-known non-monotonic perturbations in hurricane intensity within the general level of strength computed from the spawning ambient as discussed earlier.

If the basic overall physics of all computer models is the same, why is there a proliferation of programs (Ooyama 1969; Rosenthal 1970; Sundqvist 1970; Yamasaki 1968; Kurihara 1971)? The reason is that while the programs all agree with Riehl-Malkus concepts on the cyclone scale, there are physical processes that are difficult to parameterize (turbulent diffusion, radiational transfer); also, there is the question of how to model the cumulus-convection scale within the cyclone-scale program.

First, it is questionable whether it is feasible to seek a uniformly valid solution to the entire cyclone when large gradients occur over relatively small scales in important subregions of the storm (e.g., the frictional boundary layer and the eyewall), while small gradients occur over relatively large scales in the bulk of the storm (the rapidly swirling

regions and the outflow layer aloft). On current computers usually a more-or-less fixed grid of 10 km or 20 km radial resolution and at most thirteen layers of vertical resolution is adopted by practical considerations; the overwhelming bulk of the grid points then lie outside the eye, the eyewall, and the frictional boundary layer -- where important processes are occurring.* The domain sizes are quite limited (typically 440 km) so relatively weak boundary conditions (requiring only that purely advective influx occur at the side boundaries) are employed and much of the storm lies outside the domain of computation. When a closed system is studied (no radial inflow over side boundaries), the peak intensity increases monotonically with domain size, even to 1200 km (Rosenthal 1971b). Just what can be discerned from such results is mute. It seems more useful to subdivide the storm into natural portions where different gradients and phenomena are operative, as in Carrier's approach.

Next, what may be proven with current numerical models deserves consideration. The air/sea interaction postulated by Riehl and Malkus has been parameterized into Rosenthal's model and into Ooyama's, in lieu of a solution of the boundary layer. Naturally results from both models reflect this formulation and do not corroborate its physical validity, although Rosenthal would disagree (Rosenthal 1971b, p. 767, 771-772). In fact, the computation is curve-fitting in the sense that numerical techniques are rated "... on intuitive meteorological inspection of test results" (Anthes, Rosenthal, and Trout 1971, p. 747). Errors introduced by finite-differencing are used to simulate physical phenomena (as lateral mixing); the more accurate the differencing, the worse the results.† Deterioration of results

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*Even devoting four points to defining a frictional boundary layer profile is quite marginal.

†"The deterioration of the solutions with the introduction of the centered difference scheme was not anticipated, and, indeed, was quite disappointing. Not only does the less accurate upstream method provide model storms with more acceptable structure and better consistency between wind and pressure but also (sic) consistency provides an internal dissipation of kinetic energy of the same order of magnitude as the surface dissipation... It appears... that this is the correct proportionality between internal dissipation and surface dissipation... These 'beneficial' aspects of upstream differencing are clearly fortuitous... They seem to lead to the conclusion that, with our present lack of knowledge concerning the interactions (continued)

with increased precision is common in curve-fitting. It also appears that numerical models are so sensitive to small changes in initial data and in parameterizations of frictional effects (necessitated by the absence of a solution for the boundary layer) that little can be learned about intensification and, at most, only the mature-stage structure is meaningful.*

II.D IMPLICATIONS OF THE MODELS ON SEEDING

The current mode of seeding is to introduce silver iodide crystals in supercooled water believed to exist high in the eyewall, and sometimes also in rainbands both close to and far from the eyewall. The silver iodide will hopefully cause the water droplets to freeze and release the heat of fusion (Battan 1969). The proposed mechanism by which this heat release (and attendant temperature rise and density decrease) causes an amelioration of tropical cyclone intensity has been altered several times (Rosenthal 1971a; Gentry 1971b) and apparently is now uncertain [Gentry (1971a) proposes several mechanisms]; even in its original concept (Simpson and Malkus 1964), the mechanism by which seeding was to be efficacious seemed nebulous to the current authors. The one accepted point is that seeding in the nascent eye of a developing tropical storm should be avoided since this procedure would probably abet intensification (Rosenthal 1971a).

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†(continued) between the cumulus scale and the macroscale, the diffusive effects provided by upstream differencing are probably as good a representation of the statistical effect of the cumulus motions on the macroscale velocity fields as anything currently available. Such a conclusion, of course, only points to a high degree of ignorance with regard to an extremely important meteorological problem. It is by no means a solution." (Rosenthal 1970, p. 657-658).

*"The 'organizational' period is about twice as long as that found in our previously published results... This is primarily a result of replacing the constant drag coefficient (3×10^{-3}) with the variable C_D...

The time needed for the model cyclone to become organized is also highly sensitive to the arbitrary initial conditions...

The material presented in the last few paragraphs indicates that the length of the organizational period, as given by model calculations, is only of significance when experiments are compared against each other." (Rosenthal 1971, p. 769.)

Since the storm is naturally oscillating in intensity and the threat of litigation has constrained the number of seeding experiments, there is little to permit discrimination of natural and artificially induced changes in intensity. This is aggravated by the fact that even proponents of seeding anticipate only a ten-to-fifteen percent decrease in maximum winds. In the one case in which larger decreases were noted, the anomaly is now attributed to synoptic peculiarities relating to the upper-level outflow (Hawkins 1971).

The National Hurricane Research Laboratory (Gentry 1969a; Gentry 1969b; Gentry 1970) has discussed a six-to-twelve hour cycle of amelioration after seeding; physical basis for this time scale has yet to come forth. Further, if the central pressure deficit is reduced as reported, eyewall seeding must alter the eye in an as-yet unidentified manner. More complete post-seeding probing of the tropical cyclone would be helpful in evaluating these claims.

Rather similar computer models have produced different guidance with regard to current seeding practice. Sundqvist (1971) states flatly that it will intensify the hurricane. Rosenthal (1971a) suggests that it will displace the maximum winds to greater radii, reduce the maximum winds by ten-to-fifteen percent*, and increase outer winds by ten to fifteen percent. The reasons for the discrepancy are not available because some details remain unpublished.

The present authors have been, and still are, skeptical about the effectiveness of current seeding practices. If the Carrier model is valid, silver iodide seeding can only transiently upset the stable hurricane configuration. Under this model, warm-fog dispersal methods would have to be applied to the entire "throughput supply" layer of warm moist air to achieve the significant goal of premature rainout in outer spiral bands. The layer of warm moist air is so spatially extensive that such attempts seem somewhat impractical. Seeding to divert the path slightly holds

*Statistical treatment of a model suggests multimillion dollar annual savings in damage from seedings which would reduce peak winds by fifteen percent (Boyd, Howard, Matheson, and North 1971).

little better promise than seeding to alleviate intensity, since there seems no way to discern what path alterations were due to human intervention under current understanding.

III. THEORY OF TROPICAL CYCLONE INTENSIFICATION

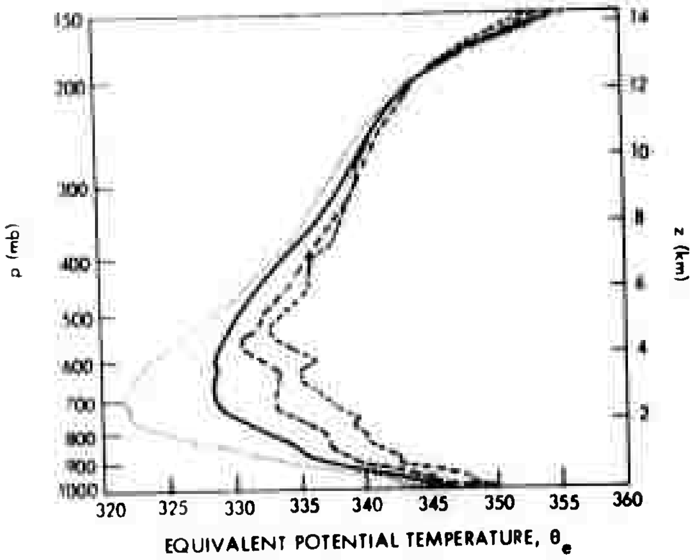
A more difficult problem than understanding the quasisteady structure of a mature tropical cyclone is developing a transient analysis explaining intensification from a weaker disturbance. The subtleties of the problem are suggested by the fact that of several hundreds of disturbances over the tropical oceans in autumn, annually but about fifty cyclones form and there is currently no way to predict which disturbances will intensify.

No complete picture of intensification will be given here. But for the Carrier model, the only one deemed worthy of further study, the outline of an intensification theory will be presented, together with several successively more sophisticated models aimed at quantitative exploration of details.

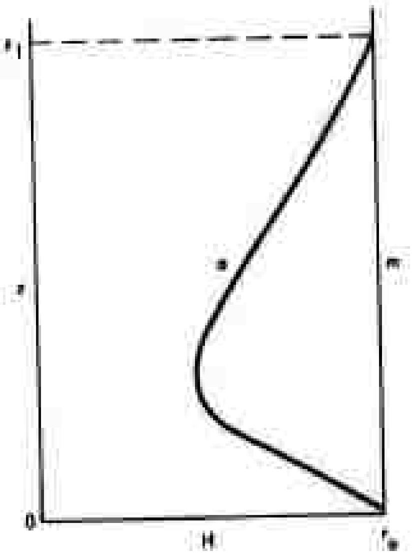
III.A CARRIER'S OUTLINE OF INTENSIFICATION

Carrier (1971b) has already sketched an intensification process by which his quasisteady mature model evolves in time from a tropical depression. Tracing back to even earlier evolution seems premature at the current state of understanding. It can hardly be overemphasized that everything which follows is either a reproduction or refinement of Carrier's earlier work.

In a tropical depression there <u>is</u> radial inflow in region I and there is weak Rankine-vortex-like swirling, the maximum azimuthal speed lying relatively far from the axis (axisymmetric model). The swirling would quickly establish a shear layer beneath it; there is weak upflux out of (downflux into) the surface layer where the swirling speed increases (decreases) with radial distance. Since linear theory correctly predicts the downflux in the mature stage with rapid swirl, linear theory certainly suffices during intensification. However, since equilibration of the boundary layer requires times of $O(\Omega^{-1})$, or roughly sixteen hours in the tropics, a transient linear theory will be required. It must yield a radial influx through the boundary layer II in excess of the radial inflow speed



THE DISTRIBUTION OF θ_e AS A FUNCTION OF PRESSURE (p) IN MILLIBARS OR HEIGHT (z) IN KILOMETERS FOR DAYS WITH FOUR CHARACTERISTIC WEATHER TYPES: AVERAGE (SOLID); SUPPRESSED CONVECTION (DOTTED); MODERATELY ENHANCED CONVECTION (LONG DASHES); AND STRONGLY ENHAVCED CONVECTION (SHORT DASHES). THE DIFFERENCES IN THE DISTRIBUTIONS RESULT PRIMARILY FROM THE VERTICAL TRANSPORT OF WATER VAPOR BY THE CONVECTIVE CLOUDS.



TOTAL ENTHALPY $H = c_pT + q^2/2 + LY + gz$ VERSUS ALTITUDE z, WHERE q IS WIND SPEED, L IS LATENT HEAT OF VAPORIZATION PER UNIT MASS AND Y IS THE MASS FRACTION OF WATER VAPOR. THE CURVE LABELLED a IS TYPICAL OF THE AMBIENT ATMOSPHERE AND m IS THE SEQUENCE OF STATES ACHIEVED BY STARTING AT SEA LEVEL AND FOLLOWING A MOIST ADIABATIC PROCESS (AS IN THE EYEWALL)

Fig. III.A.1 On the left is a plot of the equivalent potential temperature measured near Barbados (in the Lesser Antilles) in July-August 1968, showing a minimum at about 700 mb, which becomes less clearly defined as convection increases [taken with caption from Garstang, La Seur, Warsh, Hadlock, and Petersen (1970, p. 494)]. On the right are the approximations used here; the total enthalpy H, since the dynamic contribution is negligible in the ambient and never a dominant contribution, is effectively the same as $c_p\,\theta_e$. Clearly the role of enhanced convection is to alter the typical ambient profile for H in the direction of a vertical line.

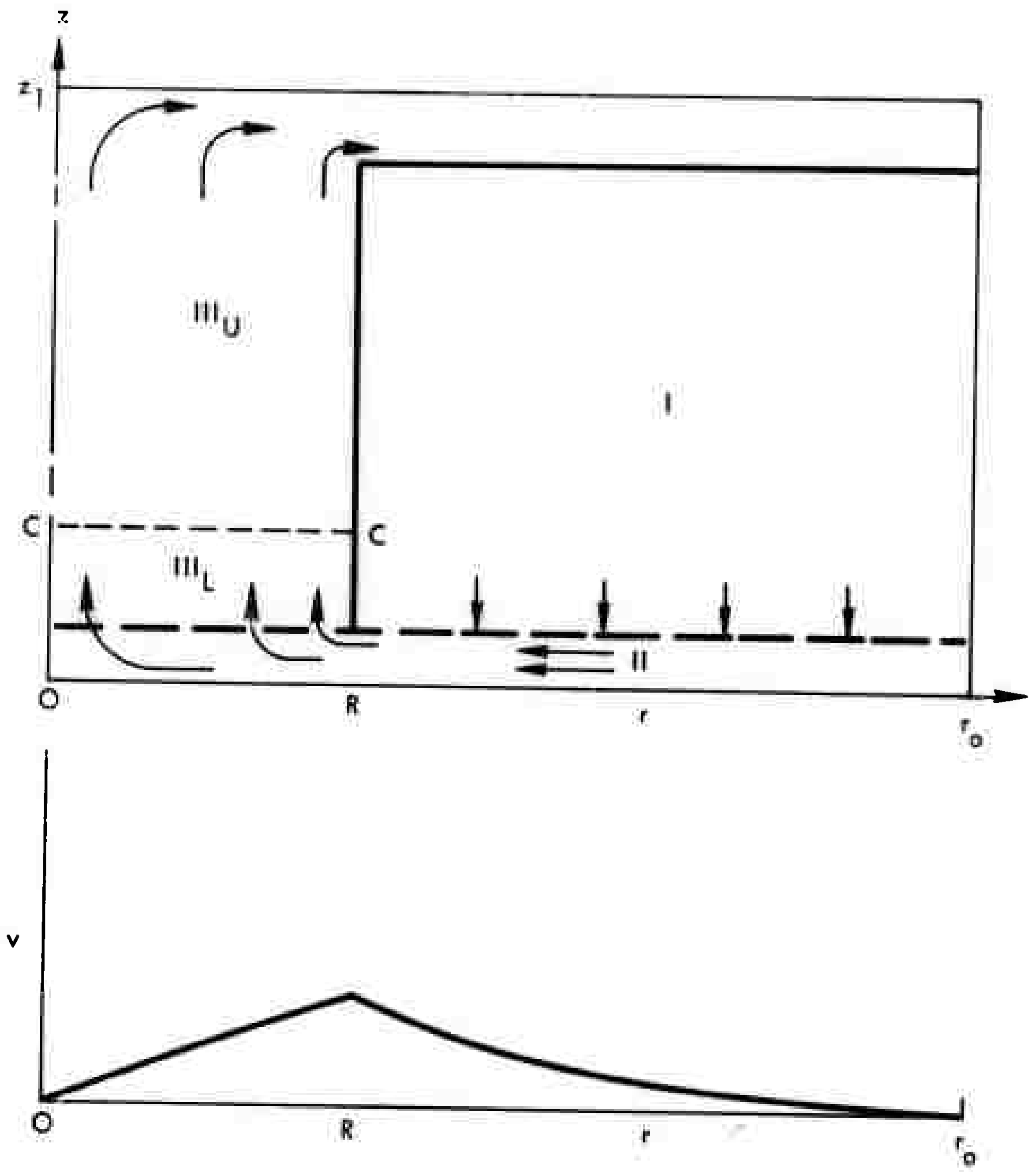


Fig. III.A.2 In this schematic view of the flow configuration and circumferential velocity distribution in an intensifying tropical depression at some early time $t = t_1$ (say), the interface C-C between the new and initial air in the core is idealized as horizontal for convenience. Meticulous details such as this are of no current concern since many points have yet to be quantitatively resolved.

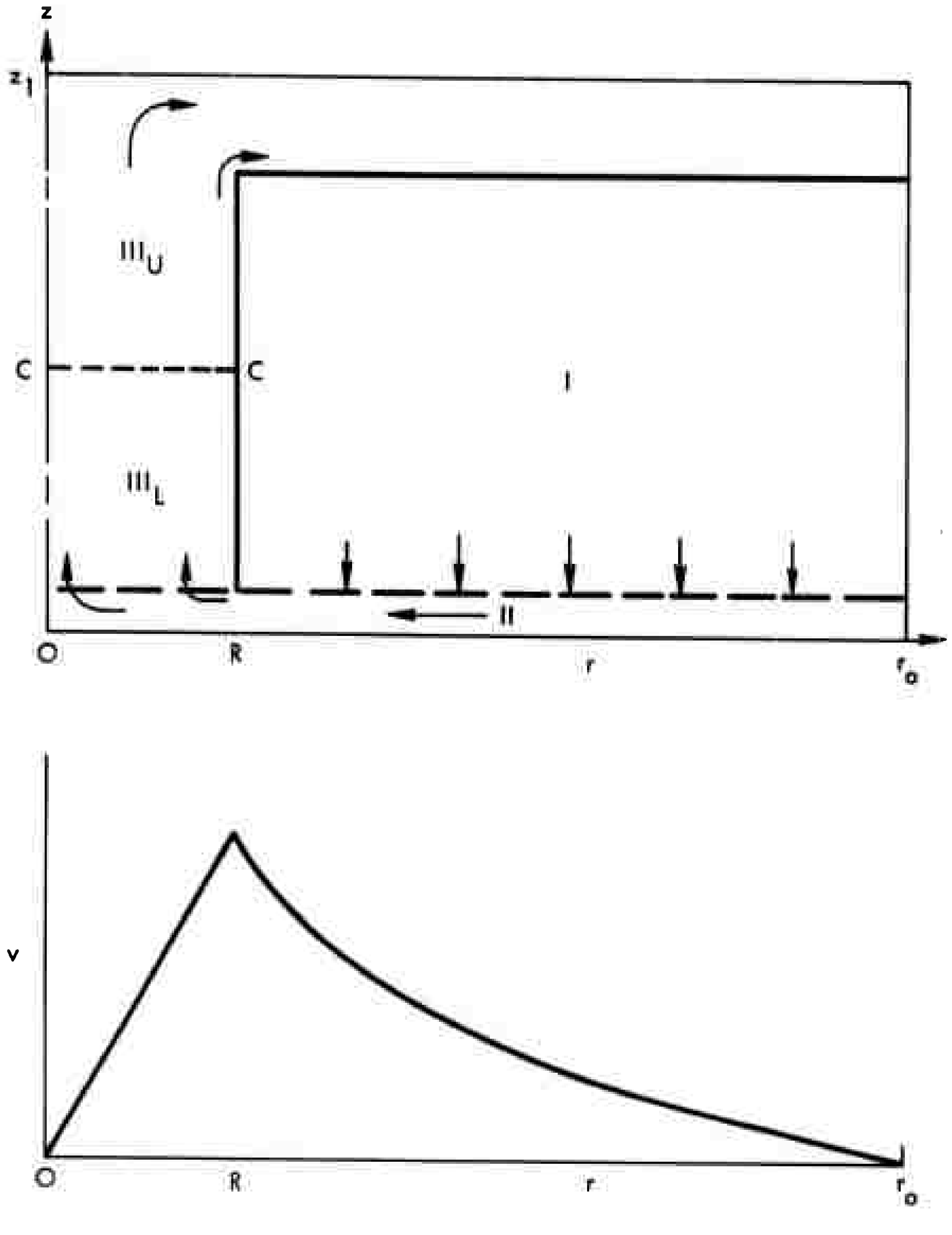


Fig. III.A.3 Intensification from tropical depression to hurricane has proceeded to a more advanced stage in this schematic diagram, holding at $t = t_2 > t_1$. The magnitude of the maximum swirl is increased and its position lies closer to the axis of symmetry.

in I. Incidentally, the times characterizing significant change in the thermodynamic state of the lower tropical ambient are probably on the order of 100 hours, a time span partly related to the eddy viscosity appropriate for the tropical atmosphere. Hence, unless the intensification time from depression to cyclone greatly exceeds 100 hours -- and this seems dubious -- the thermodynamic state of the ambient may be considered fixed throughout intensification.

The air erupting from the boundary layer begins to displace the air initially in the central core of the developing storm. The air initially present in the core is of slightly lower pressure than the ambient air at the edge of the storm, but not vastly different in vertical stratification. The air erupting from the boundary layer under the Rankine-vortex-like swirl displaces the air initially present in the core vertically upward; since there is a "lid" on top of the storm, the vertically displaced initial air is, near the top of the core, squeezed radially outward.

The following competition develops. The new air rising out of the boundary layer is drawn entirely from relatively warm moist air near the bottom of the atmosphere. Thus, displacing the air initially present in the core is relatively light air. On the other hand, the convective motion of new air is small, especially at early times, and the ambient processes (turbulent diffusion, radiational cooling, cumulus convection) try to maintain the original, near-ambient stratification in the core. If the convective displacement wins out, then the core becomes lighter and lighter, relative to a column of air at the outer edge of the storm [see Figure (III.A.1)].

Thus, the swirling in I has led to a downflux in II, a spiraling inward in the boundary layer and an upflux into the core, and a lightening of the core by hydrostatic considerations. For dynamic consistency, the centrifugal force (anticipated to be the dominant inertial effect) must increase to balance the augmented radial pressure gradient. Since angular momentum is conserved in I, where friction is negligible, the fluid particles must necessarily move in closer to the axis of symmetry (axis of rotation). The result is that in time, in the Rankine-vortex-like swirl distribution, the maximum azimuthal speed increases in magnitude and the position of the

maximum lies closer to the axis [cf. Figures (III.A.2) and (III.A.3)]. Hence, the more the pressure falls in the core, the more fluid sinks into the boundary layer to spiral inward, erupt upward, and cause further pressure reduction in the core. If the crucial early competition is resolved in favor of the organized convection, ultimately the particles erupting out of the boundary layer rise so quickly that they lie on a moist adiabat, and the greatly lightened core is entirely flushed of its original fluid.

No mention has yet been made of the eye. This is in direct contrast with the description of intensification given in Palmén and Newton (1969) in which the eye is depicted as being gradually formed as the pressure deficit develops. In the Carrier model the central core is completely flushed of ambient-like air, so that the air in the core lies on a moist adiabat based on sea-level ambient conditions, before any trace of an eye is to be found. A Rankine-vortex-like swirl holds everywhere. From this fully developed one-cell structure, a two-cell structure with a calm center region emerges rapidly, probably in much less than an hour, owing to inertial oscillation, in the following way.

As the pressure falls in the core relative to ambient, the particles in I necessarily move in closer to the axis to permit a compensating centrifugal force to develop. Once the core is flushed and moist adiabatic ascent characterizes the full height of the core, no further pressure deficit can be generated. By inertia, the spinning particles continue to move in, a dynamic imbalance is created, and a radial acceleration develops to force the particles away from the axis of symmetry. This reverse motion creates a rarefaction at the center, and relatively dry warm motionless air sinks down the axial column to form an eye. This air may be air from above the storm or rained-out, slowly swirling air entrained out of the top of the moist adiabatic column [cf. Figure (III.A.4)]. Because there is no appreciable swirl (hence no associated pressure gradient) in the eye, there is no frictional boundary layer under the eye. The moist adiabatic column becomes an annulus displaced from the axis, i.e., the eyewall; the inertial oscillations of the eyewall eventually damp in time.

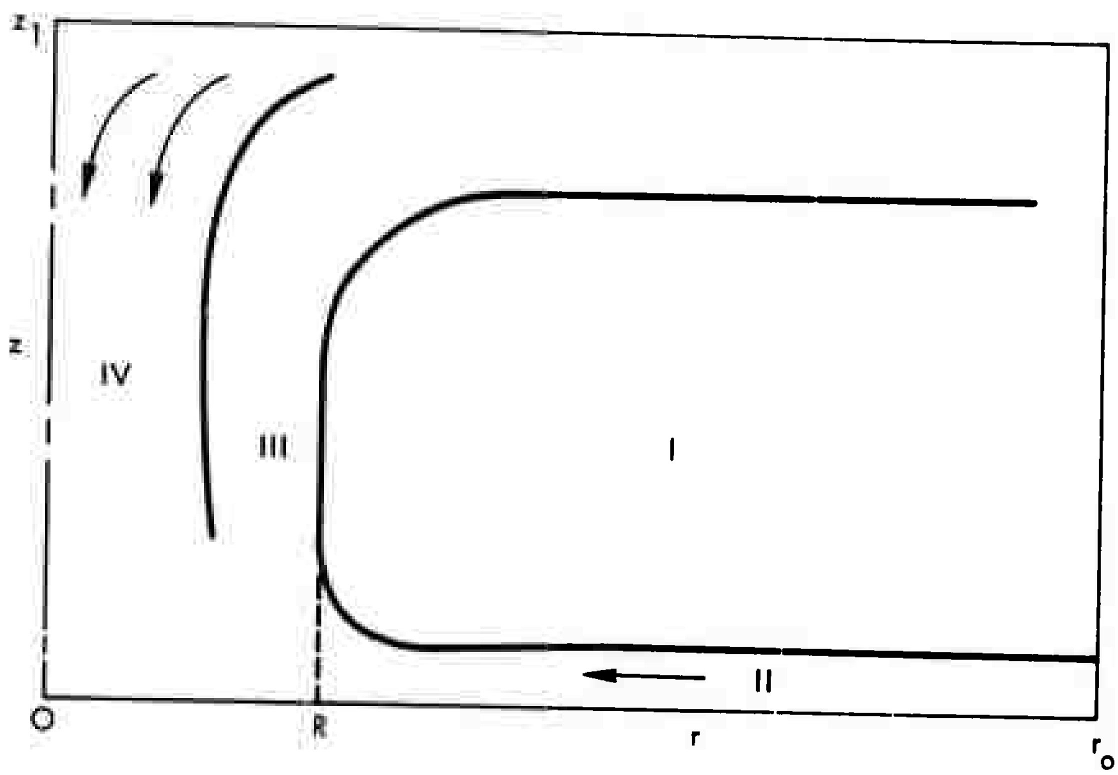


Fig. III.A.4 Schematic picture of the flow configuration which prevails when R is increasing and a nascent eye is being filled with relatively dry and motionless air, which sinks down from the top of the storm under dry-adiabatic recompression. With the formation of an eye lighter in weight than the eyewall, the terminal stages of intensification and the beginnings of quasisteady mature-stage structure are realized.